Environmental Flow Physics & Computation Group

Influence of Basset History Force on Preferential Concentration of Small Particles and Bubbles in Turbulence

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Abstract

Computation of the Basset history force can be very expensive, and hence, in past studies, (at least some fraction of) the history has been neglected even when its contribution may be important. Here the effects of the Basset force on preferential concentration (PC) of small particles in a homogeneous isotropic turbulence under a wide range of the particle-to-fluid mass density ratio γ have been investigated. The Basset force is approximated by the method of van Hinsberg et al. Compared with the traditional window method where the Basset integral is evaluated only over the latest period of size t_{win}, van Hinsberg's method that approximates the tail of the Basset force kernel by exponential functions gives a better result by using a much shorter (typically, two-order-of-magnitude smaller) t_{win}. The presence of the Basset force weakens the level of PC to some extent, especially at γ of around 1.5-10 for heavy particles and smaller than 0.7 for light particles.

Introduction

- **Maxey-Riley eq.**
  - ✓ For a small isolated rigid spherical particle in nonuniform flow

$$m_p\frac{d\mathbf{v}_p}{dt} = 6\pi a\mu(\mathbf{u}-\mathbf{v}_p) \quad \text{Stokes drag force}$$
$$+ m_f\frac{D\mathbf{u}}{Dt} \quad \text{Pressure gradient}$$
$$-(m_p-m_f)g\mathbf{e}_3 \quad \text{Gravity force}$$
$$+\frac{1}{2}m_f\left(\frac{D\mathbf{u}}{Dt}-\frac{d\mathbf{v}_p}{dt}\right) \quad \text{Added mass force}$$
$$+3\sqrt{3\mu a m_f}\int_{-\infty}^{t}\frac{\frac{d\mathbf{u}}{d\tau}-\frac{d\mathbf{v}_p}{d\tau}}{\sqrt{t-\tau}}d\tau \quad \text{Basset history force}$$

$\Longleftrightarrow$

$$\frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u}-\mathbf{v}_p}{\tau_p^*} + \beta\frac{D\mathbf{u}}{Dt} - (1-\beta)g\mathbf{e}_3 + \sqrt{\frac{3\beta}{\pi\tau_p^*}}\int_{-\infty}^{t}\frac{\frac{d\mathbf{u}}{d\tau}-\frac{d\mathbf{v}_p}{d\tau}}{\sqrt{t-\tau}}d\tau$$
$$\beta = \frac{3}{2\gamma+1},\quad \tau_p^* = \frac{3\tau_p}{3-\beta}$$

Table 1. Relation among β, $\beta^{1/2}$, and the particle-to-fluid mass density ratio γ.

γ	∞	1000	100	10	3	1	0.9	0.5	0.1	0.01	0
β	0	0.0015	0.0149	0.1429	0.4286	1	1.0714	1.5000	2.5000	2.9412	3
$\beta^{1/2}$	0	0.0387	0.1222	0.3780	0.6547	1	1.0351	1.2247	1.5811	1.7150	1.7321

Approximation to Basset force

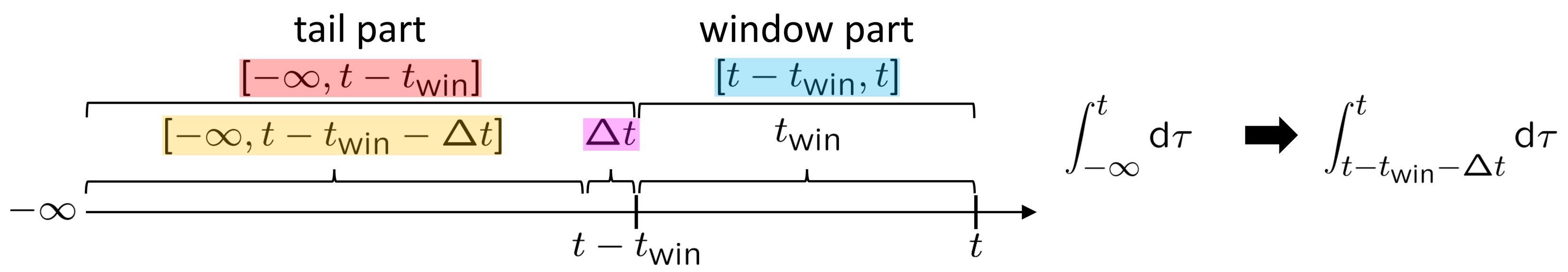
(van Hinsberg et al. JCP 230 (2011) 1465)

$$\mathbf{f}_B(t) = \sqrt{\frac{3\beta}{\pi\tau_p^*}}\int_{-\infty}^{t}\frac{\frac{d\mathbf{u}}{d\tau}-\frac{d\mathbf{v}_p}{d\tau}}{\sqrt{t-\tau}}d\tau = c'_B\int_{-\infty}^{t}\frac{1}{\sqrt{t-\tau}}\mathbf{g}(\tau)d\tau$$
$$= \underbrace{c'_B\int_{-\infty}^{t-t_{win}}\frac{1}{\sqrt{t-\tau}}\mathbf{g}(\tau)d\tau}_{\text{tail part: } \mathbf{f}_{B,tail}} + \underbrace{c'_B\int_{t-t_{win}}^{t}\frac{1}{\sqrt{t-\tau}}\mathbf{g}(\tau)d\tau}_{\text{window part: } \mathbf{f}_{B,win}}$$

- ✓ Kernel $(t-\tau)^{-1/2}$ in tail part is approximated by superposition of exponential functions:

$$\frac{1}{\sqrt{t-\tau}} \approx \sum_{i=1}^{m} a_i\sqrt{\frac{e}{t_i}}\exp\left(-\frac{t-\tau}{t_i}\right)$$

- ✓ It enables us to compute tail part of the integral in a recursive way:



$$\int_{-\infty}^{t}d\tau \;\Rightarrow\; \int_{t-t_{win}-\Delta t}^{t}d\tau$$

i.e., the yellow part of the integral can be easily calculated using the value of the orange part at the previous time step

Numerical experiment

- **Flow field**
  - ✓ Forced homogeneous isotropic turbulence at $Re_\lambda = 42$
  - ✓ Spectral DNS with 48^3 Fourier modes
- **Particles**
  - ✓ Particle-to-fluid mass density ratio γ
    - Light particles – 0, 0.001, 0.01, 0.1, 0.3, 0.5, 0.7, 0.9
    - Heavy particles – 1.005, 1.1, 3, 10, 100, 1000, 10000
  - ✓ Stokes number $S_\eta = 1.0$
  - ✓ One-way coupling, ignoring gravitational effects
  - ✓ # of particles: $1.5*48^3$, initially seeded at random locations
  - ✓ $\Delta t = 0.03\tau_\eta$, total period of simulation time $180\tau_\eta$

Window method vs. van Hinsberg's method

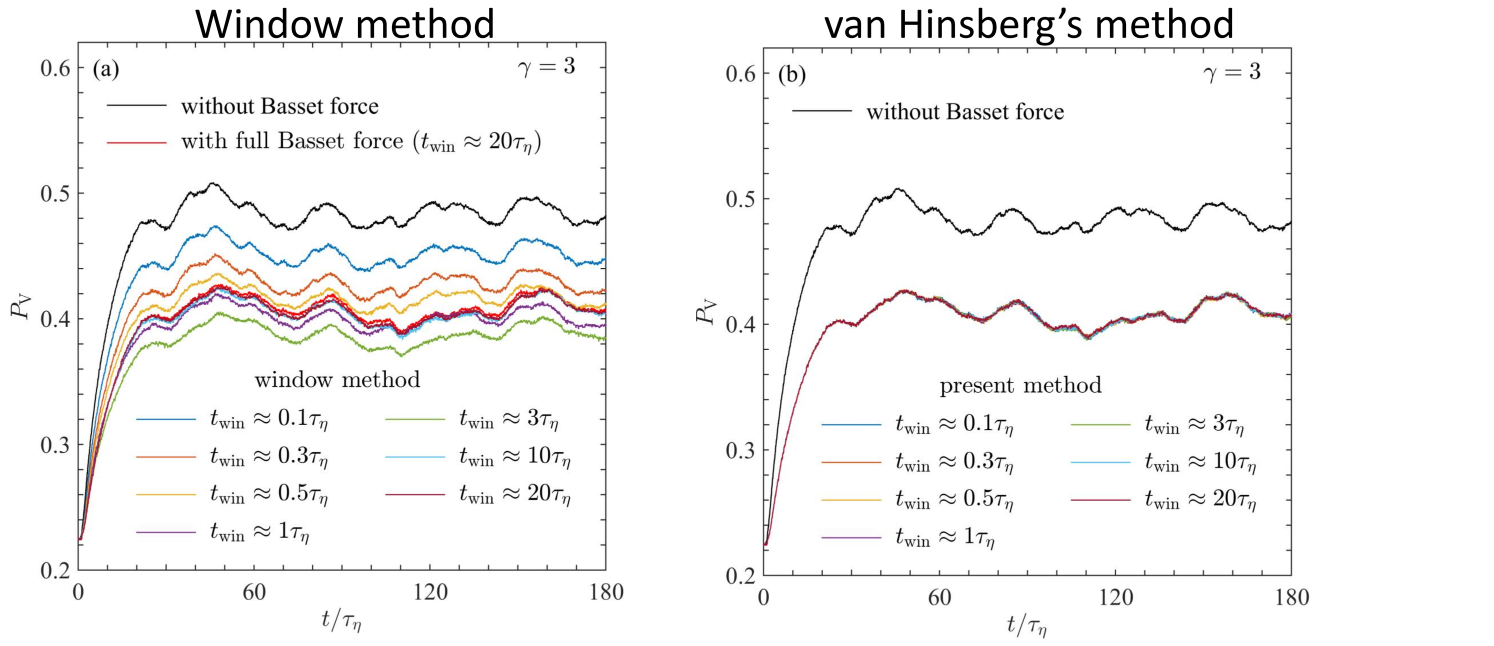


Figure 1. Influence of parameter t_{win} that defines the extent of contribution from $\mathbf{f}_{B,win}$, on the particle void fraction P_v.

Particle void fraction P_v

- represents the ratio of # of computational cells containing no particles
- describes the extent of nonuniformity in particle distribution

Impact of Basset force on PC of light/heavy particles

- **Influence on particle void fraction P_v**

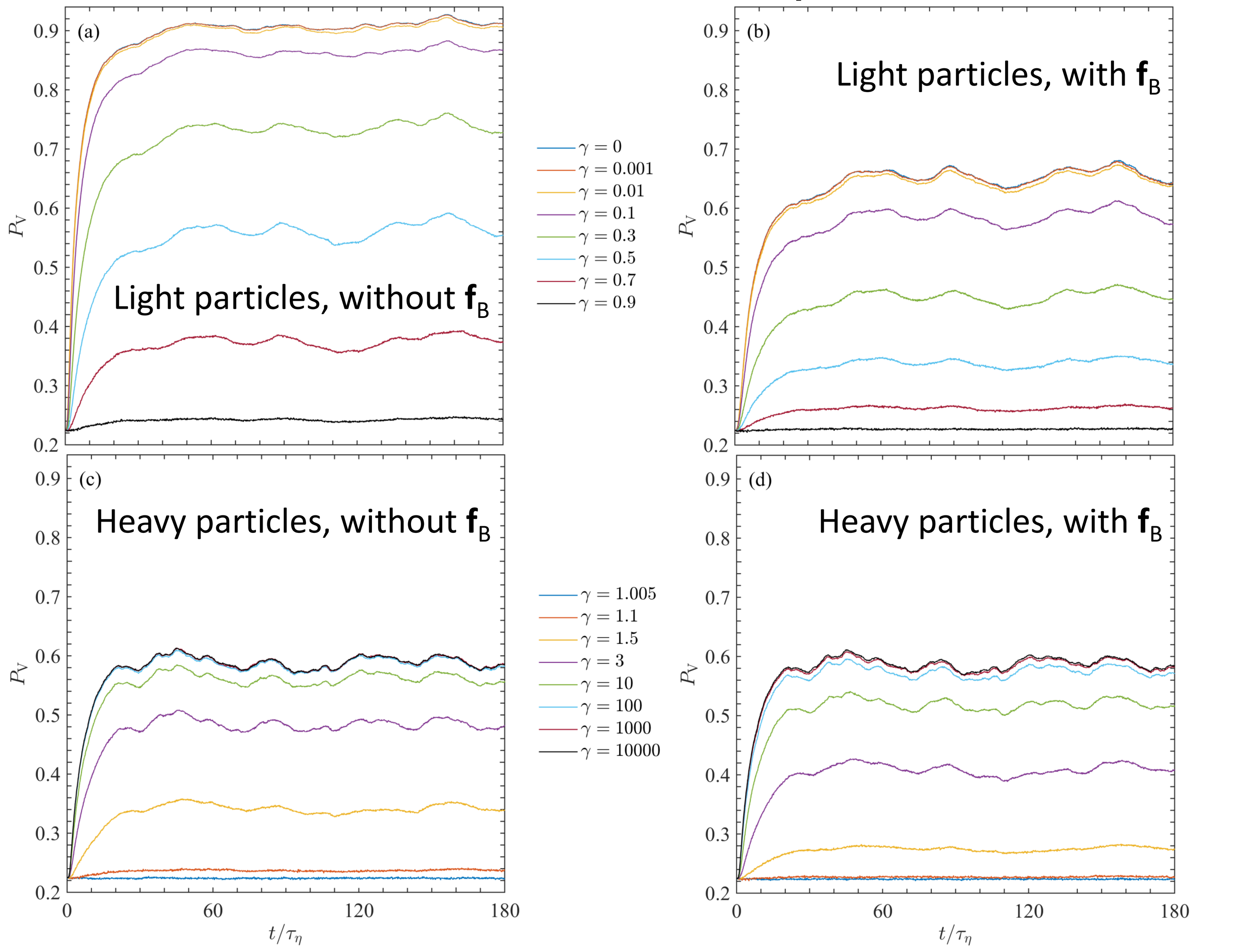


Figure 2. Influence of mass density ratio γ on P_v.

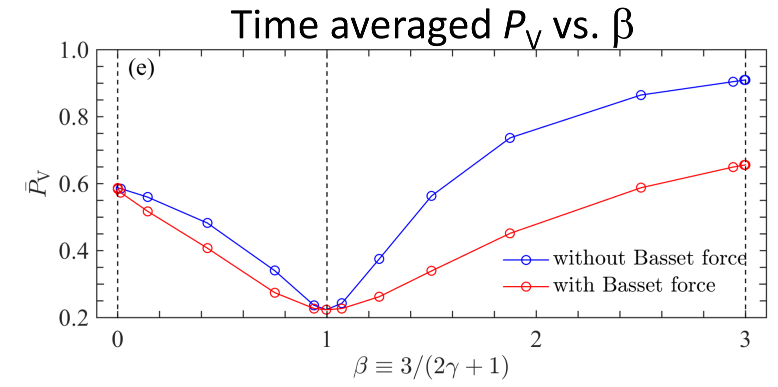


Figure 2. Continued.

- **Influence of γ on instantaneous particle distribution**

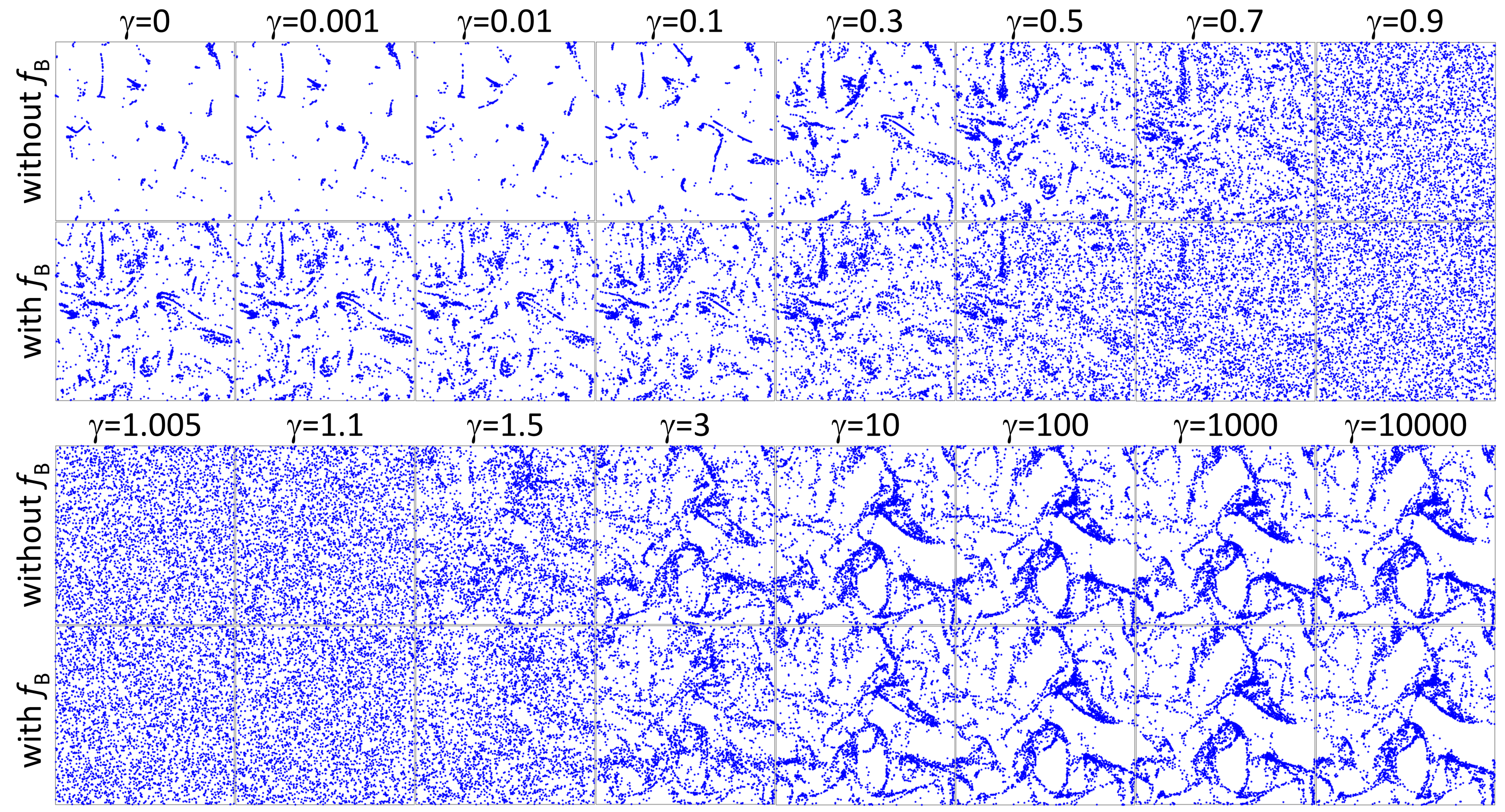


Figure 3. Influence of γ on instantaneous spatial distribution of particles inside a thin layer.

- **Relation between particle distribution and coherent vortical structures**

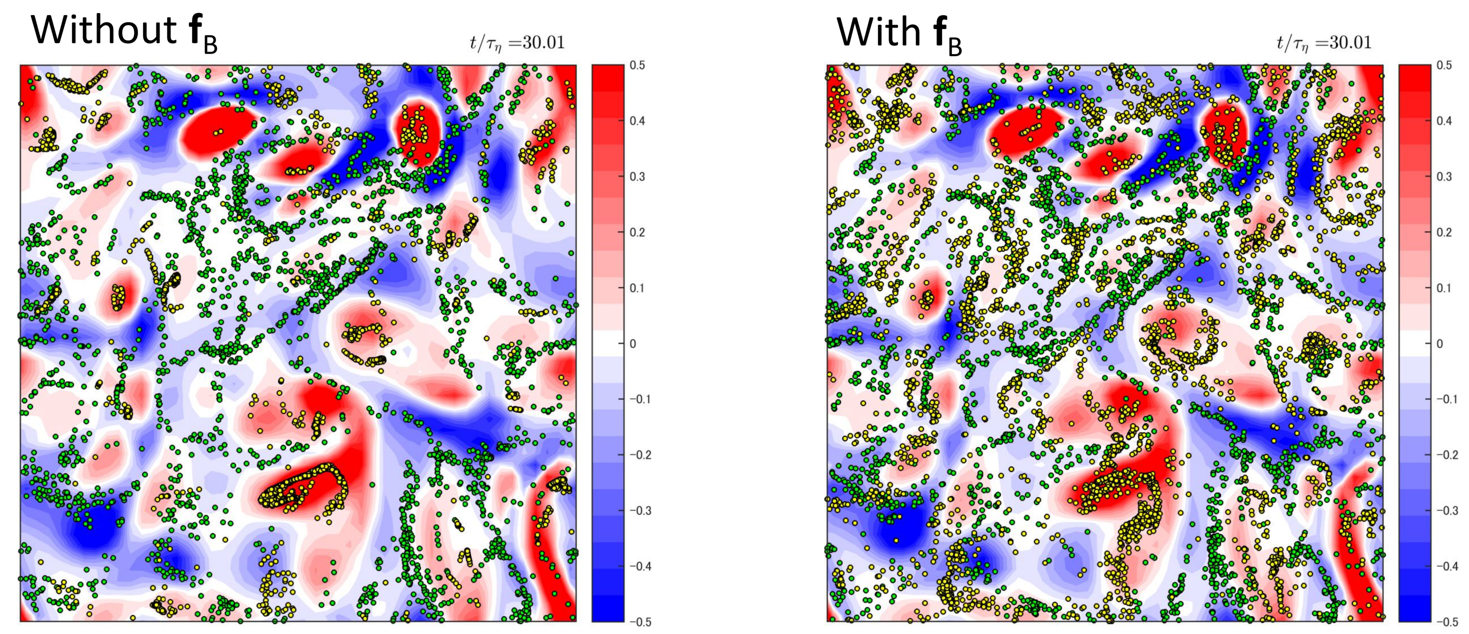


Figure 4. Relation between spatial distribution of particles (greel: very heavy particle γ = 10000; yellow: ultimately light particle γ = 0) and coherent vortical structures in homogeneous isotropic turbulence visualized by color contours of the second invariant of velocity-gradient tensor normalized by Kolmogorov scale.

Conclusion

- **Computation of Basset history force**
  - ✓ van Hinsberg's approximation is very efficient and accurate.
  - ✓ Traditional window method requires great caution in accuracy.
- **Influence of Basset history force on PC (preferential concentration)**
  - ✓ Presence of Basset force weakens PC to some extent.
  - ✓ It is especially noticeable at γ = 1.5 - 10 for heavy particles (i.e., solid particles in liquid) and smaller than 0.7 for light particles (i.e., bubbles in liquid).
  - ✓ Further investigation is necessary to clarify effects of two-way coupling and gravitational acceleration.