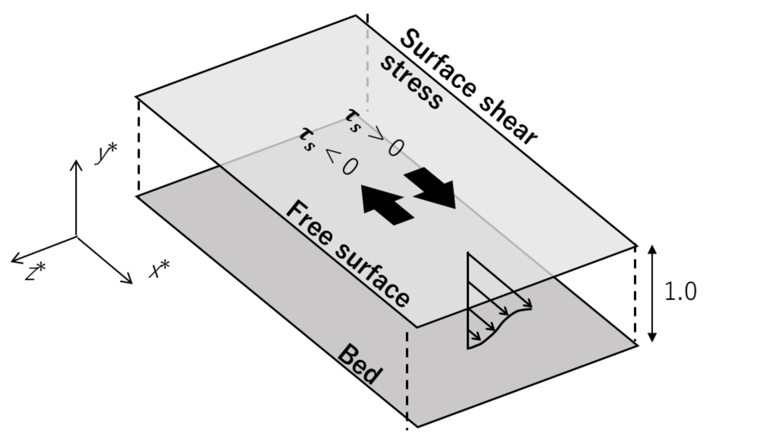
Simulating the Effect of Wind-Driven Shear Stress on Turbulent Open-Channel Flow

Shoko Sugimoto[1], Ko Shirakawa[1], Yuji Sugihara[1] ([1]Kyushu University , Japan) , Michio Sanjou[2] , Takaaki Okamoto[2] ([2]Kyoto University , Japan)

Backgorund and Objectives

Though several experimental approaches have been attempted so far, there are no clear findings or conclusions about the response of open channel turbulence to wind-driven shear stress. This problem may be redefined as an examination of the effect of wind stress on mass transport in river flow.

In this study, we investigate the effect of the surface shear stress on open-channel flow turbulence through numerical simulations. We discuss the cross-correlation coefficients between the surface scalar flux and the turbulence characteristics close to the water surface to identify turbulent motions controlling the air-water scalar transport.



$$\begin{cases} \rho g h \sin\theta + \rho u_{\tau s}^2 = \rho u_{\tau b}^2 \\ \dfrac{\rho g h \sin\theta}{\rho u_{\tau b}^2} = 1 - \dfrac{u_{\tau s}^2}{u_{\tau b}^2} = 1 - \tau_s^* \\ \tau_s^* \equiv \dfrac{u_{\tau s}^2}{u_{\tau b}^2} < 1 \end{cases}$$

The gravitational and the surface shear forces are balanced with the bottom shear force.

Direct Numerical Simulation

The basic equations consist of the continuity equation for incompressible fluid, the Navier-Stokes equation and the advection-diffusion equation for a passive scalar as follows: The subscript $*$ denotes the dimensionless quantity.

$$\begin{cases} \dfrac{\partial u_i^*}{\partial t^*} + u_j^* \dfrac{\partial u_i^*}{\partial x_j^*} = -\dfrac{\partial p^*}{\partial x_i^*} + \dfrac{1}{Re_\tau}\left(\dfrac{\partial^2 u_i^*}{\partial x_j^* x_j^*}\right) + \dfrac{\delta_{1i}}{Fr_\tau^2} \\ \dfrac{\partial u_i^*}{\partial x_i^*} = 0, \quad \dfrac{\partial c^*}{\partial t^*} + u_j^* \dfrac{\partial c^*}{\partial x_j^*} = \dfrac{1}{ScRe_\tau}\dfrac{\partial^2 c^*}{\partial x_j^* \partial x_j^*} \\ \dfrac{1}{Fr_\tau^2} \equiv \dfrac{gh\sin\theta}{u_{\tau b}^2} = 1 - \tau_s^*, \ Re_\tau \equiv \dfrac{u_{\tau b} h}{\nu}, \ Sc \equiv \dfrac{\nu}{D} \end{cases}$$

k-ε Model

The time difference is the forward difference and the spatial difference is the Crank-Nicolson method. The solution of the difference equation used a tridiagonal matrix. $(\bar{U}, \bar{k}, \bar{\varepsilon})$

Reynolds equation

$$\left(\frac{C_\mu h U_\delta}{\kappa \delta u_{\tau b}}\right)\frac{\partial \tilde{U}}{\partial \tilde{t}} = (1-\tau_s)\left(\frac{\kappa u_{\tau b}}{C_\mu U_\delta}\right) + \frac{\partial}{\partial \eta}\left(\left(\tilde{\nu}_t + \frac{1}{Re_\tau}\frac{C_\mu h}{\kappa \delta}\right)\frac{\partial \tilde{U}}{\partial \eta}\right)$$

Turbulent energy transport equation

$$\left(\frac{\kappa U_\delta}{\sqrt{C_\mu} u_{\tau b}}\right)\frac{\partial \tilde{k}}{\partial \tilde{t}} = \left(\frac{\kappa}{C_\mu^{3/4}}\right)^2 \frac{\partial}{\partial \eta}\left(\left(\frac{\tilde{\nu}_t}{\sigma_k} + \frac{1}{Re_\tau}\frac{C_\mu h}{\kappa \delta}\right)\frac{\partial \tilde{k}}{\partial \eta}\right) + \left(\frac{\kappa U_\delta}{\sqrt{C_\mu} u_{\tau b}}\right)^2 \tilde{\nu}_t \left(\frac{\partial \tilde{U}}{\partial \eta}\right)^2 - \tilde{\varepsilon}$$

Transport equation of turbulent energy dissipation rate

$$\left(\frac{\kappa U_\delta}{\sqrt{C_\mu} u_{\tau b}}\right)\frac{\partial \tilde{\varepsilon}}{\partial \tilde{t}} = \left(\frac{\kappa}{C_\mu^{3/4}}\right)^2 \frac{\partial}{\partial \eta}\left(\left(\frac{\tilde{\nu}_t}{\sigma_\varepsilon} + \frac{1}{Re_\tau}\frac{C_\mu h}{\kappa \delta}\right)\frac{\partial \tilde{\varepsilon}}{\partial \eta}\right) + \left(\frac{\kappa U_\delta}{\sqrt{C_\mu} u_{\tau b}}\right)^2 C_1 \frac{\tilde{\varepsilon}}{\tilde{k}}\tilde{\nu}_t \left(\frac{\partial \tilde{U}}{\partial \eta}\right)^2 - C_2 \frac{\tilde{\varepsilon}^2}{\tilde{k}}$$

Wall function

$$U_\delta = u_{\tau b}\left[\frac{1}{\kappa}\ln\left(Re_\tau\left(\frac{\delta}{h}\right)\right) + A_r\right]$$

$$k_\delta = \frac{u_{\tau b}^2}{\sqrt{C_\mu}}, \quad \varepsilon_\delta = \frac{u_{\tau b}^3}{\kappa \delta}$$

Eddy viscosity coefficient

σ_k=1.0, σ_ε=1.3, C_1=1.44, C_2=1.92, C_μ=0.09, k=0.41 : A_r=5.3:

Boundary condition

$$\tilde{U}=1, \ \tilde{k}=1, \ \tilde{\varepsilon}=1 \ \text{at} \ \eta = \frac{\delta}{h}$$

$$\left(\tilde{\nu}_t + \frac{1}{Re_\tau}\frac{C_\mu h}{\kappa \delta}\right)\frac{\partial \tilde{U}}{\partial \eta} = \left(\frac{C_\mu h u_{\tau b}}{\kappa \delta U_\delta}\right)\tau_s$$

$$\frac{\partial \tilde{k}}{\partial \eta} = 0, \quad \tilde{\varepsilon} = \frac{(\delta/h)}{(y'/h)}\tilde{k}^{3/2} \ \text{at} \ \eta = 1$$

Damping function f_s

$$\tilde{\nu}_t = C_\mu f_s(\eta)\frac{\tilde{k}^2}{\tilde{\varepsilon}}$$

$$f_s(\eta) = 1 - \exp\left(-A_s \frac{h\varepsilon_\delta}{k_\delta^{3/2}}(1-\eta)\right) = 1 - \exp(-B_s(1-\eta))$$

Cross-Correlation Coefficients

$$C_x(r_x) = \frac{\langle F^{*\prime}(x^*,z^*) A^{\prime}(x^*+r_x, z^*)\rangle}{F_{rms}^* A_{rms}^*} \quad (x^*\text{: steamwise direction})$$

$$C_z(r_z) = \frac{\langle F^{*\prime}(x^*,z^*) A^{\prime}(x^*, z^*+r_z)\rangle}{F_{rms}^* A_{rms}^*} \quad (z^*\text{: spanwise direction}) .$$

r_x and r_z denote the distances in each direction from (x^*, z^*), respectively. Also, $F^{*\prime}(x^*,z^*)$ and $A^{*\prime}(x^*,z^*)$ are the fluctuation components of the surface flux and turbulent quantities at (x^*,z^*), indicating $F^{*\prime}(x^*,z^*) = F^*(x^*,z^*) - \langle F^*(x^*,z^*)\rangle$ and $A^{*\prime}(x^*,z^*) = A^*(x^*,z^*) - \langle A^*(x^*,z^*)\rangle$

$$F^* = \frac{1}{Re_\tau Sc}\frac{\partial C^*}{\partial y^*}\bigg|_{surface} \quad \text{(Downward is positive)}$$

$$\beta^* = \left(\frac{\partial u_1^*}{\partial x_1^*} + \frac{\partial u_3^*}{\partial x_3^*}\right)_{surface}$$

Streamwise vorticity $\quad \omega_x = \dfrac{\partial w^*}{\partial y^*} - \dfrac{\partial v^*}{\partial z^*}$

Spanwise vorticity $\quad \omega_z = \dfrac{\partial v^*}{\partial x^*} - \dfrac{\partial u^*}{\partial y^*}$

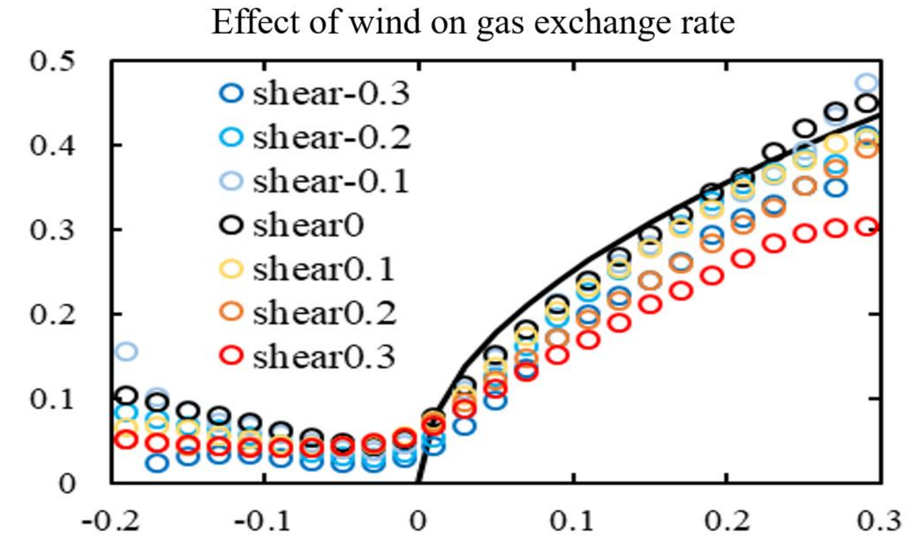
Analytical Solution for Scalar Transport Velocity

Relation of scalar transport velocity with surface divergence: Analytical solution and DNS ($Re_\tau = 150$)

$$k^+ = \sqrt{\frac{2}{\pi}} Sc^{-1/2}\sqrt{\beta^+}$$

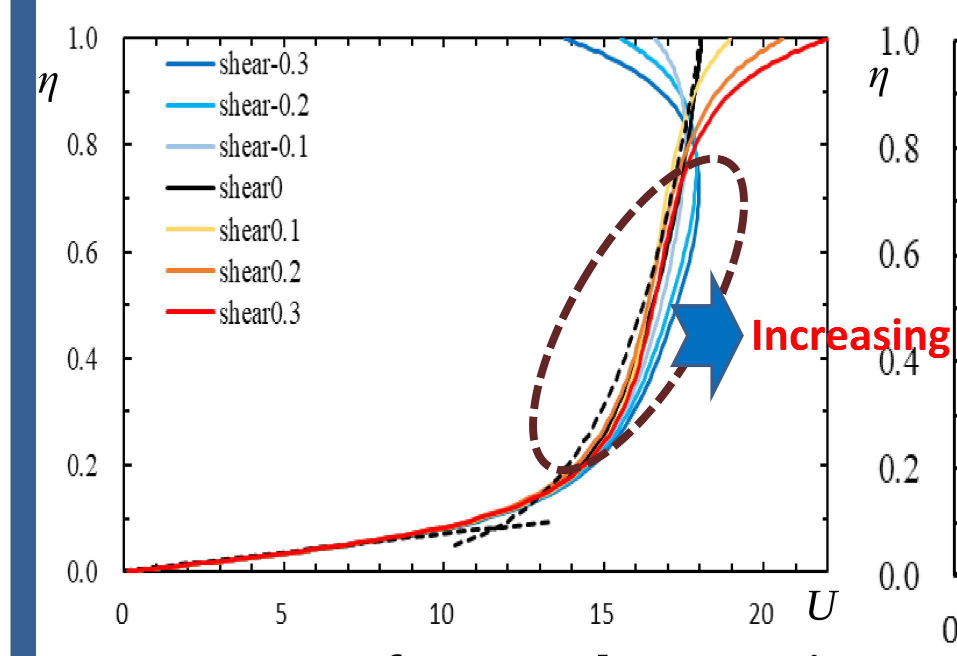
+ indicates the dimensionless quantity
k^+: scalar transport velocity
β^+: surface divergence

Effect of wind on gas exchange rate

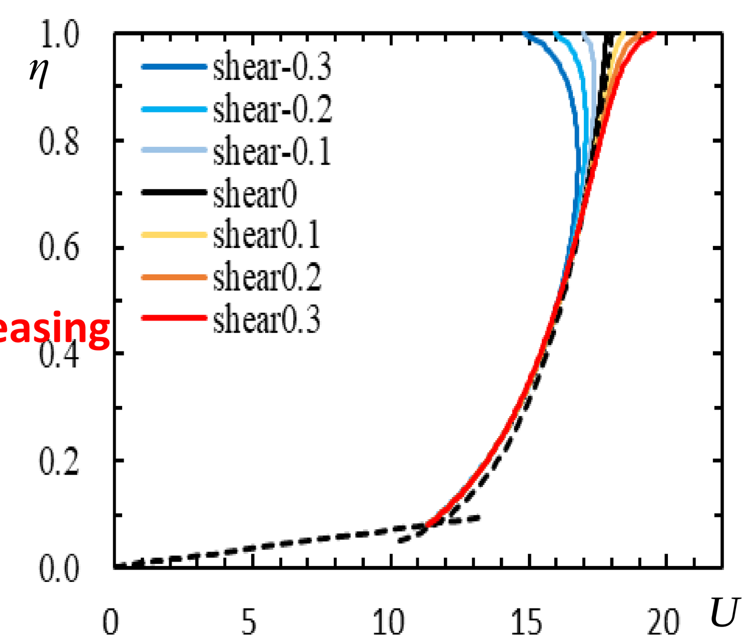


Results and Discussion

1 DNS, $Re_\tau = 150$

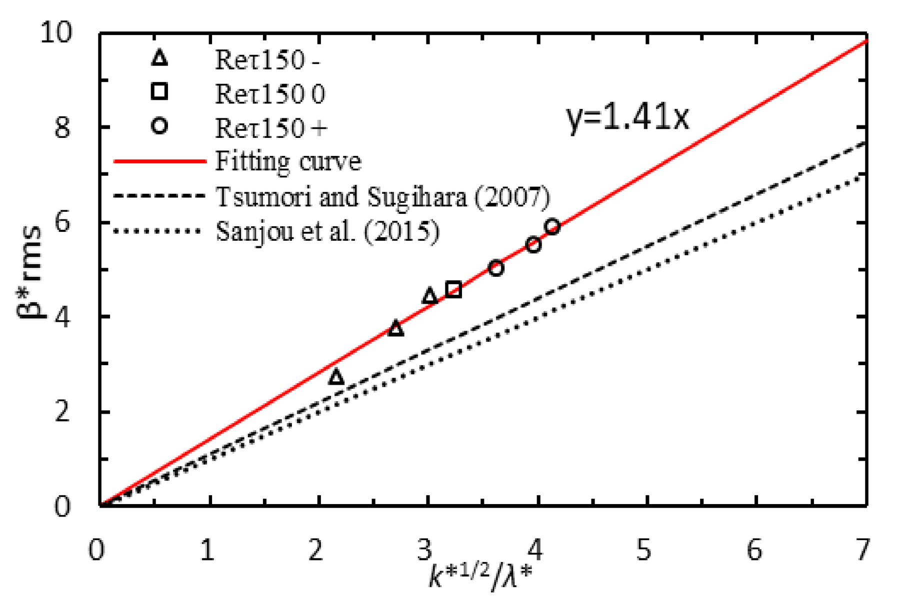


2 k-ε model, $Re_\tau = 150$



Response of averaged streamwise velocity distributions to τ_s

3 Scaling relation for surface divergence based on Taylor micro scale (DNS, $Re_\tau = 150$)

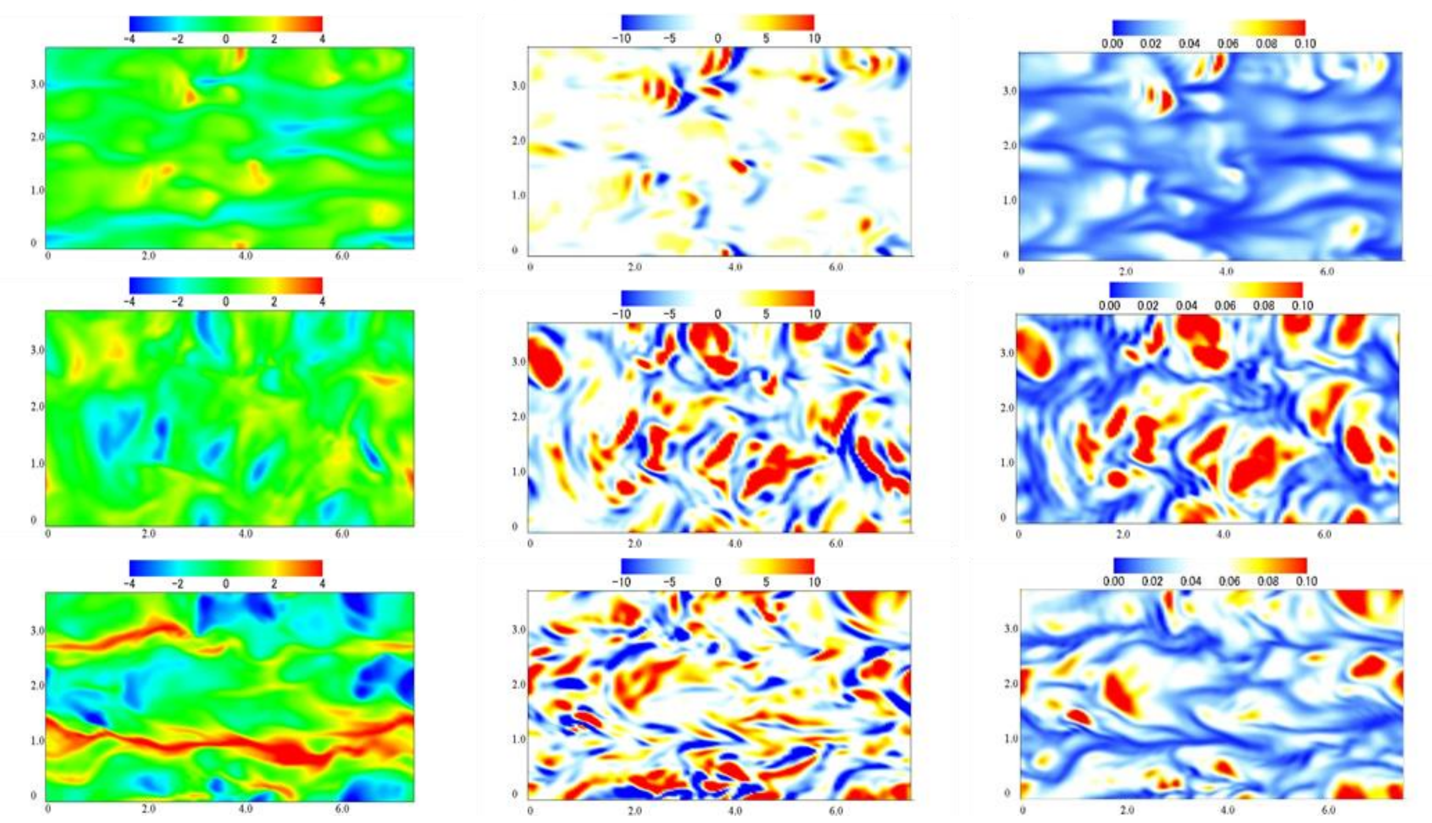


Streamwise velocity fluctuation on water surface , Surface divergence , Scalar flux on water surface

1. **In the case of $\tau_s^* = 0$, the scalar flux increases intensively in the positive surface divergence areas.**
2. **Effects of surface shear stress**
   - The degree of the agreement is found to be low between the surface divergence and the scalar flux.
   - The relation of the surface divergence with the scalar flux is changed in comparison with the analytical solution.
   - The streched structures can be seen in the near the water surface under the action of positive shear stress.

Turbulence dynamics governing the scalar transport seems to change from the surface divergence to the other physical mechanism.

Streamwise velocity fluctuation on water surface (DNS, $Re_\tau = 150$) | Surface divergence | Scalar flux on water surface



Cross correlation for streamwise direction , Cross correlation for spanwise direction

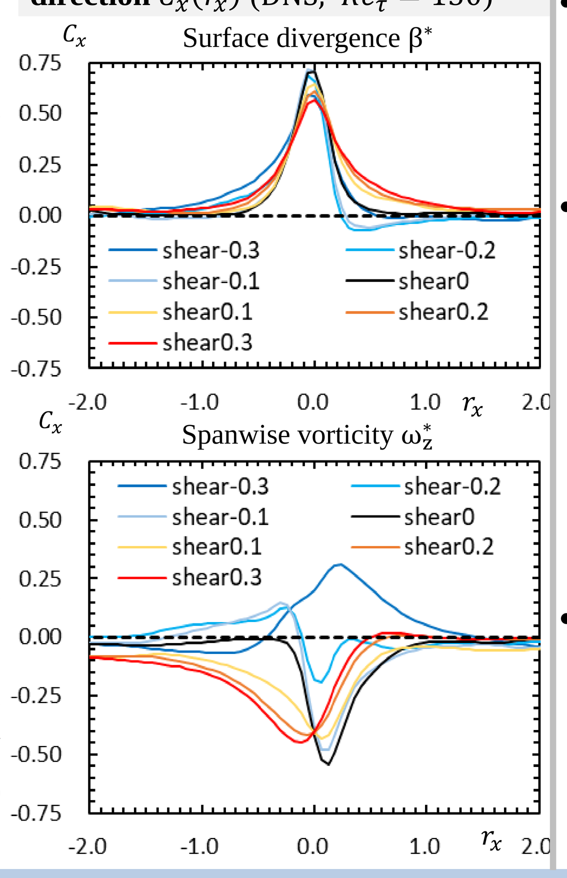
Cross correlation for surface divergence

- The case of τ_s^*=0 is symmetric with respect to the origin of $r_x = 0$, whereas for the other values of τ_s^*, it becomes asymmetric.
- When the positive shear stress, $C_x(r_x)$ increases relatively in the region of $r_x > 0$. Conversely, in the cases of $\tau_s^* = -0.1$ and -0.2, the downward overshoot of $C_x(r_x)$ is observed in the region of $r_x > 0$.

Cross correlation for ω_z^*

- From the streamwise correlation with $\omega z*$ shown in the figure, it is seen that the relation of $Cx(rx)$ with $\tau s*$ becomes complicated especially in the case of the negative shear stress.

Cross correlation for streamwise direction $C_x(r_x)$ (DNS, $Re_\tau = 150$)



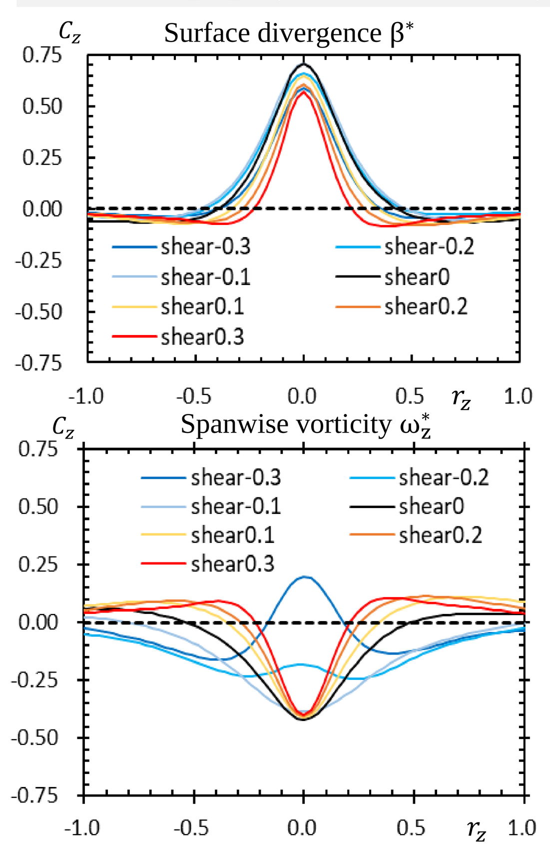
Cross correlation for surface divergence

- The cross-correlation coefficients $C_z(r_z)$ shows symmetric shapes with respect to $r_z = 0$ regardless of the sign and magnitude of τ_s^*.
- The maximum values of the cross-correlation coefficients $C_x(r_x)$ and $C_z(r_z)$ take 0.75 approximately when $\tau_s^* = 0$, supporting that the surface divergence is an important physical quantity for describing the scalar flux at the water surface.

Cross correlation for ω_z^*

- The correlation drawn in the figure shows symmetrical variation with respect to $r_z = 0$, but the behavior is not monotonous in the case of negative shear stress.

Cross correlation for spanwise direction $C_z(r_z)$ (DNS, $Re_\tau = 150$)



Conclusion

- ✓ **In this study, we have investigated the response of open-channel turbulence to wind-driven surface shear stress by using the numerical methods of DNS and the standard k-ε model.**
- ✓ It has been seen that the vertical distributions of the streamwise velocity and the Reynolds stress vary depending on the positive or negative sign of the surface shear stress.
- ✓ **The numerical results from DNS have provided that under the condition of negative shear stress, the streamwise velocity around the half-water depth is increased than that in the case of no shear stress.**
- ✓ **The surface divergence calculated from DNS** has been confirmed to be universally **scaled with the Taylor microscale** regardless of the positive or negative sign of the shear stress.
- ✓ **It has been concluded from the cross-correlation coefficients that the scalar flux considerably increases when vortex structure is arranged so as to induce a strong upward flow toward the water surface.**

References

1. Sanjou, M. and Nezu, I. (2011). Turbulence Structure and Coherent Vortices in Open-Channel Flows with Wind-Induced Water Waves, *Environmental Fluid Mechanics*, Vol.11, pp.113-131.
2. Sanjou, M., Takahashi, K., Toda, K. (2015). Hydrodynamic Effects of Strip Roughness on Surface Velocity Divergence in Open-Channel Turbulent Flows, *Journal of Japan Society of Civil Engineers*, Ser. B1 (Hydraulic Engineering) Vol.71, No.1, pp.1-10. (In Japanese)
3. Tsumori, H. and Sugihara, Y. (2007). Lengthscales of motion that control the air-water gas transfer in grid-stirred turbulence, *J. Marine Systems*, Vol. 66, pp.6-18.