

## SUSPENDED MOVEMENT OF POLLUTANT AND SEDIMENT IN OPEN CHANNEL FLOW AS STOCHASTIC PROCESS

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### ABSTRACT

The Stochastic Theory has shown that the sediment particles 2-D trajectories  $\omega(x, z, t)$  result from the combination of random 1-D series of movement periods: (i) an alternating series of downstream longitudinal displacements intercalated by time periods when the particle does not move in this direction or sense; (ii) an alternating series of descendent vertical displacements intercalated by periods when the grain does not move towards the bed. These series are defined by Mobility Functions:  $\lambda_{x1}$ ;  $\lambda_{z1}$ ;  $\lambda_{x2}$  and  $\lambda_{z2}$ , which analytical expressions describe the instantaneous and punctual movements of single particles. Thus, the bed and suspended load are both described by the same Stochastic Theory. The Objectives of this article are to present: (1) the 2-D longitudinal and vertical models; (2) the experimental devices used for the study of bed and suspended longitudinal movements with vertical transfers; and (3) the importance of temporal and spatial intensity functions, in the Stochastic Processes models' creation. Experimental results obtained in open channel flows with radioactive and fluorescent tracers were used, where the mobile bed layer behaves: (i) as a reflective barrier of fine sediments of varying concentrations and of different sizes; (ii) as a source for bed load movements of uniform sand; (iii) as a barrier of sediment absorption; and (iv) as a simultaneous barrier of absorption and reflection of sediments. A software **SPICON - Stochastic Processes with Instantaneous and Continuous injections** was developed to study the Homogeneous Poisson Models.

*Keywords:* Movement of sediments and pollutants, Stochastic Processes, Radioactive and Fluorescent Tracers, SPICON.

### 1. INTRODUCTION

For several years, the authors among others (Einstein, 1937; Wilson-Jr, 1972, 1987, 2012; Vukmirović, 1975; Vukmirović and Wilson-Jr, 1976, 1977; Todorović et al, 1976; Hanno, 1979; Wilson-Jr and Vukmirović, 1981; Monteiro and Wilson-Jr, 2002, 2003; Monteiro, 2004; Wilson-Jr and Monteiro, 2016, 2019) have been dedicated to the development and applications of random models in laboratory channels with sediment labeled with radioisotopes, in streams and in rivers with fluorescent and radioactive tracers, simulating pollutants and sediments, respectively. The studies performed by these authors are summarized in the Figure 1.

The Theory of Stochastic Processes proposes a kinematic analysis of the movements of the liquid and solid phases, while considering the turbulent characteristics of the flow. Thus, problems related to: (i) the nonlinearity of the equations; (ii) the complexity of liquid and solid interactions; and, (iii) the lack of knowledge of the mutual interference of the movements of the two phases; are circumvented.

#### 1.1 Intensity of Grain Mobility Functions

The mathematical development that characterize the sediment movement as stochastic processes can be followed through the works of Wilson-Jr (1972, 1987), Vukmirović (1975), Hanno (1979) and Monteiro (2004). The authors elaborated a synthesis of this development, which are periodically updated, for improvements and applications of the theme (Wilson-Jr and Monteiro, 2016, 2019).

The bed and suspended load movements of sediment and contaminant particles in open channels flows characterize stochastic process, where the elementary events are the single grains' trajectories. They are dependent of the turbulent hydrodynamic process. These trajectories or achievements of the single particles (as shown in Figures 2 and 3) or of the group of particles are analyzed by Lagrangean or Spatial, and Eulerian or Temporal Descriptions.

Two stochastic processes are considered:

$$\vec{R}(t, \omega) = [X(t, \omega), Y(t, \omega), Z(t, \omega)] = X_{ti}(\omega); i = 1, 2, 3 \quad (1)$$

that characterizes the evolution of the particle's position vector as a function of time, which longitudinal, lateral and vertical components are  $X(t, \omega)$ ,  $Y(t, \omega)$  and  $Z(t, \omega)$ , respectively. The second 3-D stochastic process:

$$T(x, y, z, \omega) = [T(x, \omega), T(y, \omega), T(z, \omega)] = T_{xi}(\omega); \quad i = 1, 2, 3 \quad (2)$$

characterizes the particle's passing time by the point of coordinates  $(x, y, z)$ .  $T(x, \omega)$ ,  $T(y, \omega)$  and  $T(z, \omega)$  represent the times spent by the particle to travel the distances  $0x$ ,  $0y$  and  $0z$ , respectively, and  $\omega$  represents the trajectory or the sediment particle achievements.

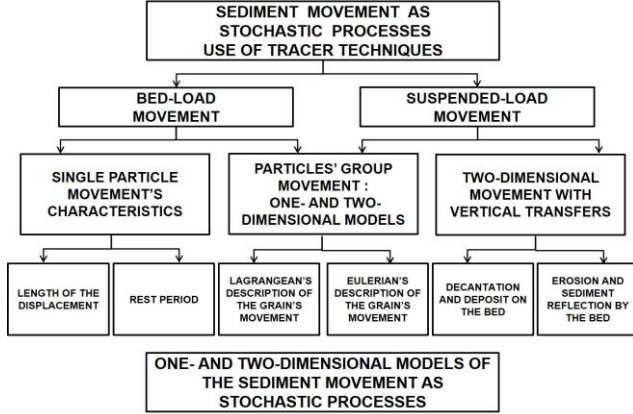
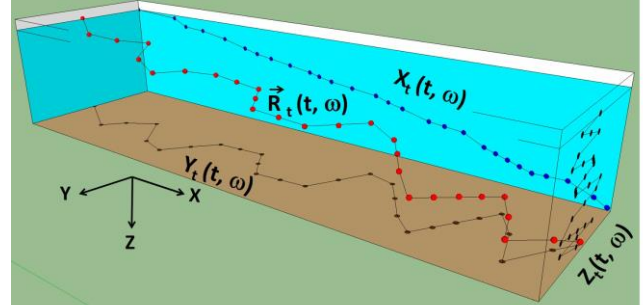


Figure 1. Studies performed with tracers on the sediment movement by the Theory of Stochastic Processes (Wilson-Jr and Monteiro, 2019)



$$\vec{R}_t(t, \omega) = [X(t, \omega), Y(t, \omega), Z(t, \omega)] = X_i(t, \omega)$$

$$T(x_i, \omega) = [T(x, \omega), T(y, \omega), T(z, \omega)]; \quad i = 1, 2, 3$$

Figure 2. Trajectories of a suspended sediment grain: elementary events of stochastic processes:

$$\vec{R}_t(t, \omega) \text{ and } T_{xi}(\omega); \quad i = 1, 2, 3$$

(Wilson-Jr and Monteiro, 2016)

$X_{ii}(\omega)$  and  $T_{xi}(\omega)$  processes can be defined by their Probability Distribution Functions:

$$F_t(x_i) = P\{X_i(t, \omega) \leq x_i; \quad x_i \geq 0; \quad i = 1, 2, 3 \quad (3)$$

$$Q_{xi}(t) = P\{T(x_i, \omega) \leq t; \quad t \geq 0; \quad i = 1, 2, 3 \quad (4)$$

which are related to each other by Todorović's Equation (5) (Todorović et al. 1966):

$$F_t(x_i) = 1 - Q_{xi}(t); \quad x_i \geq 0; \quad i = 1, 2, 3 \quad (5)$$

It was shown that the Probability Distribution Function of these random processes can be expressed in terms of two pairs of Approximate Functions  $F_{t1}(x_i)$  and  $F_{t2}(x_i)$ ;  $Q_{x1i}(t)$  and  $Q_{x2i}(t)$ ;  $i = 1, 2, 3$ , respectively, such that:

$$0 \leq F_{t1i}(x_i) \leq F_{ti}(x_i) \leq F_{t2i}(x_i) \leq 1; \quad i = 1, 2, 3 \quad (6)$$

$$0 \leq Q_{x2i}(t) \leq Q_{xi}(t) \leq Q_{x1i}(t) \leq 1; \quad t \geq 0; \quad i = 1, 2, 3 \quad (7)$$

In each direction, e. g. in the longitudinal direction  $0x_{i=1}$ , where  $x_i = x_1 = x$ , the Approximate Distribution Functions  $F_{ij}(x)$  and  $Q_{xj}(t)$ ,  $j = 1, 2$  can be explained as functions of two new stochastic processes  $G_n^{0,x}$  and  $E_n^{0,t}$  from the same elementary events  $\omega$ :

$$G_n^{0,x} = \{\mu_{0,x} = n\} \quad (8)$$

which represents the medium number of grain displacements,  $\mu_{0,x}$  over the distance  $[0, x]$ , and,

$$E_n^{0,t} = \{\eta_{0,t} = n\} \quad (9)$$

the medium number of grain displacements,  $\eta_{0,t}$  over the time period  $[0, t]$ .  $G_n^{0,x}$  and  $E_n^{0,t}$  are Markovian Processes with similar properties. So, for the set  $G_n^{0,x}$ , it has:

$$\begin{cases} P\{G_1^{x,x+\Delta x} | G_k^{0,x}\} = \lambda_2(x, k) \Delta x + \vartheta(\Delta x) \\ P\{G_v^{x,x+\Delta x} | G_k^{0,x}\} = \vartheta(\Delta x), \quad v \geq 2 \\ P\{G_0^{x,x+\Delta x} | G_k^{0,x}\} = 1 - \lambda_2(x, k) \Delta x + \vartheta(\Delta x) \\ P\{G_0^{0,0}\} = 1 \end{cases} \quad \text{when } \Delta x \rightarrow 0 \quad (10)$$

where  $\vartheta(\Delta x)$  is a grain first order infinitesimal displacement distance. The  $G_n^{0,x}$  and  $E_n^{0,t}$  occurrence probabilities are solutions of the system of equations derived from these properties:

$$\begin{cases} \frac{\partial}{\partial x} P\{G_k^{0,x}\} = \lambda_2(x, k-1) P\{G_{k-1}^{0,x}\} - \lambda_2(x, k) P\{G_k^{0,x}\} \\ \frac{\partial}{\partial x} P\{G_0^{0,x}\} = -\lambda_2(x, 0) P\{G_0^{0,x}\} \end{cases} \quad (11)$$

with the following initial conditions:

$$x = 0 \quad \begin{cases} P\{G_0^{0,x}\} = 1 \\ P\{G_k^{0,x}\} = 0; k \geq 1 \end{cases} \quad (12)$$

Similar analytical expressions to the Equations (10), (11) and (12) are obtained for the  $E_n^{0,t}$  process. The solution of these differential equations yields the probability laws for the numbers of displacements in time and spatial intervals. Two functions  $\lambda_1(t, n)$  and  $\lambda_2(x, n)$  appear, which describe the sediment particle mobility, in time and in that particular direction  $x_{i=1} = x_1 \approx x$ . Considering the three directions of the orthogonal axes  $0x_i$ ,  $i = 1, 2, 3$ , three pairs of Mobility Functions  $\lambda_{1i}(t, n)$  and  $\lambda_{2i}(x_i, n)$  are obtained, which describe the sediment grains 3-D movements, in time and space. In each  $x_i$  direction it has been:

$$\begin{cases} \lambda_{1i}(t, n) = \lim_{\Delta t \rightarrow 0} \frac{P\{E_{1i}^{t, t+\Delta t} | E_{ni}^{0,t}\}}{\Delta t} \\ \lambda_{2i}(x_i, n) = \lim_{\Delta x_i \rightarrow 0} \frac{P\{G_{1i}^{x_i, x_i+\Delta x_i} | G_{ni}^{0,x_i}\}}{\Delta x_i} \end{cases} \quad i = 1, 2, 3 \quad (13)$$

The general expressions for  $\lambda_{1i}$  and  $\lambda_{2i}$ ,  $i = 1, 2, 3$ ; were obtained by Vukmirović (1975) and Wilson-Jr (1987, 2012) considering the bedload movement of single grains labeled with radiotracers. They considered the mobility of the particle as a function of time, of the distance traveled in one direction and of its past performance in time ( $n$ ) and distance ( $k$ ), in each direction ( $i$ ):

$$\begin{cases} \lambda_{1i}(t, n) = \lambda_{1i}(t) \lambda_{2i}(n) \\ \lambda_{2i}(x, k) = \lambda_{2i}(x) \lambda_{2i}(k) \end{cases} \quad i = 1, 2, 3 \quad (14)$$

where:  $\lambda_{1i}$  and  $\lambda_{2i}$  are the particle mobility factors in each time  $t$  in the direction  $i$ , and in a certain position  $x_i$ , respectively.

These functions are obtained from experiments performed with liquid and solid particles of tracers: radioactive, dyes and chemicals, in bedload and/or suspended-load movements. Wilson-Jr (1987, 2012) classified the Stochastic Models according to their mobility functions in Homogeneous Poissonian Models (constant mobilities), No-Homogenous and No-Poissonian, which are indicated in Figures 3 and 4.

With the experimental device shown in Figures 5 and 6, the movements of suspended sediment and of bedload, with vertical transfers, were recorded. Thus, we obtained a collection of original data for the study of random movements 1-D and 2-D of sediments, cohesive and non-cohesive, and analysis of the evolution of a group of particles that moves sometimes suspended in the middle of the liquid phase, sometimes by dragging on the movable bed of a river. Particularly, for the case of suspended movement, the grain mobility functions in the longitudinal and vertical directions assume constant values and the resultant 2-D Stochastic Models are also Homogeneous Poissonian.

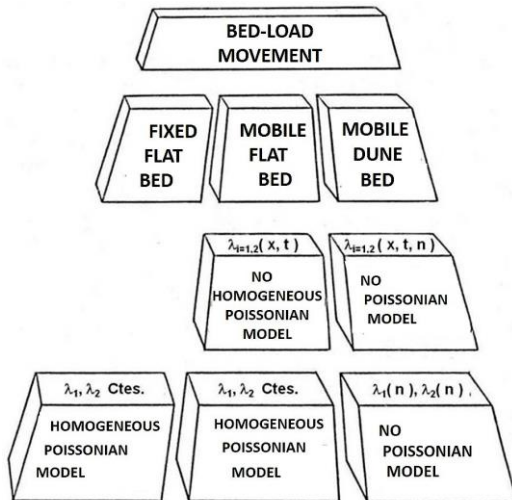


Figure 3. Stochastic models of the bedload movements (Wilson-Jr, 1987)

$\lambda_1(t, n)$	$\lambda_2(x, m)$	Authors	Date	Movement	Models	
$\lambda_1 = \text{Cte}$	$\lambda_2 = \text{Cte}$	Einstein	1937	Bed-load	Homogeneous Poissonian	
		Sayre e Hubbell	1965	Bed-load		
		Todorović e Vukmirović	1966	Bed-load		
		Paintal	1971	Bed-load		
		Todorović, Simmons, Li	1976	Suspension		
		Hanno	1980	Bed-load		
		Wilson-Jr.	1987	Bed & Suspension Load		
		Mendes e Wilson-Jr.	1995	Suspension		
		Monteiro e Wilson-Jr.	2005	Suspension		
		Wilson-Jr.	2007	Bed & Suspension Load		
$\lambda_1(t)$	$\lambda_2(x)$	Shen e Todorović	1971	Bed-load	No Homogeneous Poissonian	
		Frickmann e Wilson-Jr.	1996			
$\lambda_1 = \text{Cte}$	$\lambda_2(x, r_2)$ $r_2 > 0, \text{Cte}$	Sayre e Yang	1968	Bed-load		
		Grigg	1971			
$\lambda_1(x, r_1)$ $r_1 > 0, \text{Cte}$	$\lambda_2(x, r_2)$ $r_2 > 0, \text{Cte}$	Wilson-Jr.	1972	Bed-load		
$\lambda_1(t)$	$\lambda_2(t)$	Vukmirović e Wilson-Jr.	1975,77	Bed-load		No Poissonian
$\lambda_1(n)$	$\lambda_2(m)$					Causality Functions
? $\lambda_1(t)$	? $\lambda_2(x)$	Hung e Shen	1976	Bed-load		No Poissonian
$\lambda_1(t)$	$\lambda_2(t)$	Cheong e Shen	1983	Bed-load		No Poissonian
		Vukmirović e Wilson-Jr.	1985,87			

Figure 4. Models resulting from expressions of  $\lambda_1(t, n)$  and  $\lambda_2(x, m)$  (Wilson-Jr, 2012)

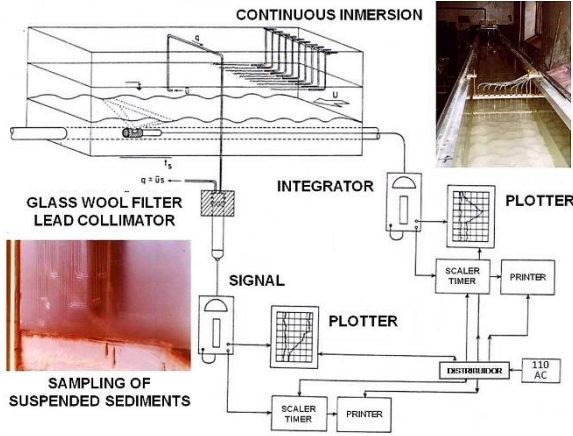


Figure 5. Immersion and detection system of bed and suspended movements of fine sediments in a laboratory channel. (Wilson-Jr, 1987)

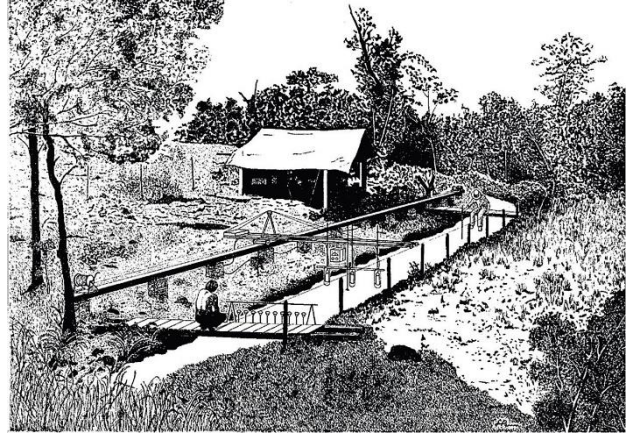


Figure 6. Experiments carried out in streams with sediments labeled with radioisotopes. (Wilson-Jr, 1987)

## 2. OBJECTIVES

The main objectives of this article are to present: (1) the 2-D longitudinal and vertical models; (2) the experimental devices used for the study of bed and suspended longitudinal movements with vertical transfers; and (3) the importance of temporal and spatial intensity functions, in the Stochastic Processes models' creation.

## 3. 1-D HOMOGENEOUS POISSONIAN MODELS

When the probability of the sediment grain make a displacement in the time interval  $[t, t+\Delta t]$  or in the space  $[x, x+\Delta x]$  does not depend, neither on the time, nor on the distance traveled, nor on the number of previous displacements, it is said that the particle has no memory and the values of the mobility intensity are constant in time and space:  $\lambda_1(t, n) = \lambda_1$  and  $\lambda_2(x, n) = \lambda_2$ .

Equations (10), (11) and (12) are simplified and the probabilities of occurrence of the sets  $G_n^{0,x}$  and  $E_n^{0,t}$  are obtained by recurrence:

$$P\{G_n^{0,x}\} = \frac{(\lambda_2 x)^n}{n!} e^{-\lambda_2 x} \quad (15)$$

$$P\{E_n^{0,t}\} = \frac{(\lambda_1 t)^n}{n!} e^{-\lambda_1 t} \quad (16)$$

that is, expressions of Poissonian probabilities with constant parameters. Therefore, the models are called Homogeneous Poissonian Models. From Poisson's Law: (i)  $1/\lambda_1$  is the average duration of a period of non-displacement (rest period for bedload longitudinal movement); (ii)  $1/\lambda_2$  is the average distance traveled by the particle during a positive displacement, and (iii)  $u_p = \lambda_1/\lambda_2$  is the average transport speed of sediment or pollutant particles (Table 1).

Wilson-Jr (1987, 2012) compared the mobilities of sediment grains of the bed and in suspension, and found that the values of the average displacements ( $1/\lambda_2$ ) are of the same order of greatness, while the average duration of the periods of non-displacement or rest ( $1/\lambda_1$ ) is much larger in the bedload than in the suspended movement, which explains its lower mobility.

For the applications of the Homogeneous Poissonian Models, Wilson-Jr and Monteiro (2019) developed one software called SPICON - Stochastic Processes with Instantaneous and CONTinuous injections, which analyzes the Eulerian and Lagrangean movements of pollutants, in suspension and of the mobile bed layer. Among other properties, the program calculates the statistical characteristics of the Stochastic Processes that describe the movement of the groups of particles, such as the probability density functions of the grain positions over time (and their moments), as illustrated in the Figure 7.

The experiments in the Paraíba do Sul River aimed the determination of the solute pollutants transport and dispersion, in a stretch between Volta Redonda and Barra do Piraí cities, in the Rio de Janeiro State. Four campaigns were performed, determining the transit time's curves of dyes concentration across six sections, distant 2.6 to 39.6 km from the injection section.

Table 1. Mobility of fine sediment grains in the bed and suspension. Homogeneous Poisson Models (Wilson-Jr, 1987, 2012)

Types of Essays	Tracer	Authors (Essays' Date)	1/λ <sub>2</sub> (cm)	1/λ <sub>1</sub> (s)	u <sub>p</sub> = λ <sub>1</sub> /λ <sub>2</sub> (cm/s)	Reynolds Number
Suspended Load	Dyes	Sayre and Chang (1968)	13.0	0.55	23.6	1.22 E05
			17.0	0.49	34.7	2.83 E05
			19.0	0.39	48.7	0.89 E05
Bedload	Radioactive (Simple Particles)	Wilson-Jr. and Vukmirović (1971-1973)	27.0	264.0	0.10	0.90 E05
			34.0	480.0	0.07	1.06 E05
			32.0	456.0	0.07	1.31 E05
			26.0	456.0	0.06	1.54 E05
			37.0	570.0	0.07	1.89 E05
Bedload	Radioactive (Group of Particles)	Wilson-Jr. and Vukmirović (1971-1973)	8.0	263.0	0.03	1.25 E05
			56.0	1500.0	0.04	1.11 E05
			63.0	2045.0	0.03	1.14 E05

- 1/λ<sub>2</sub> = average distance of a grain displacement.
- 1/λ<sub>1</sub> = average period of a non-displacement.
- u<sub>p</sub> = λ<sub>1</sub>/λ<sub>2</sub> = average velocity of the sediment transport.

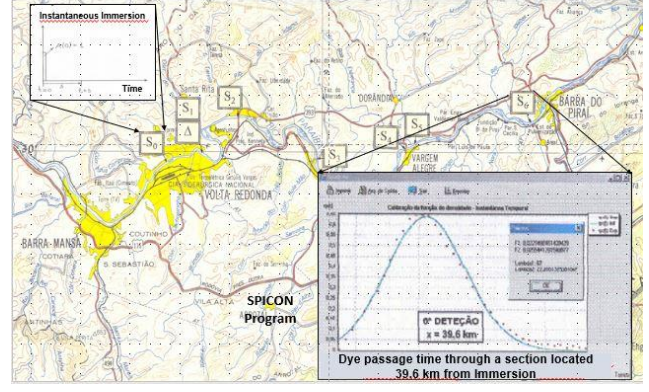


Figure 7. Experiments performed in the Paraíba do Sul River: Instantaneous and continuous dye immersions (Rhodamine). State of Rio de Janeiro, Brazil.

#### 4. 2-D HOMOGENEOUS LAGRANGEAN STOCHASTIC POISSONIAN PROCESS

After several successful 1-D applications, Wilson-Jr and Monteiro (2016, 2019) have devoted to studies of the 2-D movements in suspension - longitudinal and vertical - of cohesive and no-cohesive sediments, whose data have been obtained in the prismatic channel of the LCHF (Figure 5): 12.0 m long, 40.0 cm wide and 60.0 cm deep, and in nature (Figure 6).

For the suspended movement, the grain mobility functions in the longitudinal and vertical directions assume constant values and the resultant models are Homogeneous Poissonian. So, their expressions are:

$$\begin{cases} \lambda_{1i}(t, n) = \lambda_{1i} = \text{const.} \\ \lambda_{2i}(x_i, k) = \lambda_{2i} = \text{const.} \end{cases} \quad i = 1 \text{ and } 3 \quad (17)$$

which means that the probability of the grains' displacements, in time [t, t+Δt], and distance [x, x+Δx], Δt and Δx tending to zero, are independent of time, particle position and of previous displacements, i.e., independent of the particle history. This movement is called out of memory.

The Density Probability Functions of the particles in time t is given by equations which characterize the Homogeneous Poissonian Random Processes (Wilson-Jr and Monteiro, 2016, 2019):

$$f_t(x, z) = \frac{\partial^2 F_t(x, z)}{\partial x \partial z} = f_t(x) f_t(z) \quad (18)$$

$$0.0 \leq f_{t1}(x_i) \leq f_t(x_i) \leq f_{t2}(x_i) \leq 1.0 \quad i = 1, 3 \quad (19)$$

$$\begin{cases} f_{t1}(x_i) = \lambda_{2i} e^{-\lambda_{1i}t - \lambda_{2i}x_i} \sum_{k=0}^{\infty} \frac{(\lambda_{1i}t)^k}{k!} \frac{(\lambda_{2i}x_i)^k}{k!} \\ f_{t2}(x_i) = \lambda_{2i} e^{-\lambda_{1i}t - \lambda_{2i}x_i} \sum_{k=0}^{\infty} \frac{(\lambda_{1i}t)^{k+1}}{(k+1)!} \frac{(\lambda_{2i}x_i)^k}{k!} \end{cases} \quad i = 1, 3 \quad (20)$$

Wilson-Jr and Monteiro (2019) presented analytical expressions of the approximate statistical properties of the stochastic process  $X_i(x, z)$ , for the cases of instantaneous and continuous immersions. Among them are the Probability Density Functions  $f_{ij}(x, z)$ ; the Probability Distribution  $F_{ij}(x, z)$ , the equations of the Average Position  $M_{ij}(x, z)$  and the Variance of the Positions of the particles  $[(S_{txzj})^2]$ , for  $j = 1, 2$ . As an example, the analytical expressions of the Approximate Probability Density Functions  $f_{ij}(x, z)$ ,  $j = 1, 2$  are presented, for the cases of instantaneous and continuous immersions, in the free surface of the flow, respectively. The originality is the substitution of the inferior and superior approximations of the Probability Density Functions  $f_{ij}(x, z)$ ,  $j = 1, 2$  and other approximate statistical characteristics, by the expressions of their average values,  $f_{tm}(x, z)$ ,  $h_{tm}(x, z)$ ,  $F_{tm}(x, z)$  and  $H_{tm}(x, z)$  for example, for cases of instantaneous and continuous immersions, respectively, as shown as following:

##### 4.1 Instantaneous Immersion Case

Mean Probability Density Function  $f_{tm}(x, z)$

$$f_t(x, z) \cong f_{tm}(x, z) \cong \frac{1}{2}[f_{t1}(x, z) + f_{t2}(x, z)] \quad (21)$$

$$\begin{cases} f_{t1}(x, z) = \lambda_{21} \lambda_{23} e^{-\lambda_{11}t - \lambda_{21}x} e^{-\lambda_{13}t - \lambda_{23}z} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(\lambda_{11}t)^n (\lambda_{21}x)^n (\lambda_{13}t)^k (\lambda_{23}z)^k}{n! n! k! k!} \\ f_{t2}(x, z) = \lambda_{21} \lambda_{23} e^{-\lambda_{11}t - \lambda_{21}x} e^{-\lambda_{13}t - \lambda_{23}z} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(\lambda_{11}t)^{n+1} (\lambda_{21}x)^n (\lambda_{13}t)^{k+1} (\lambda_{23}z)^k}{(n+1)! n! (k+1)! k!} \end{cases} \quad (22)$$

Mean Probability Distribution Function  $F_{tm}(x, z)$

$$F_{t1}(x, z) \leq \left\{ F_t(x, z) \cong F_{tm}(x, z) \cong \frac{1}{2} [F_{t1}(x, z) + F_{t2}(x, z)] \right\} \leq F_{t2}(x, z) \quad (23)$$

$$\begin{cases} F_{t1}(x, z) = e^{-\lambda_{11}t - \lambda_{21}x} e^{-\lambda_{13}t - \lambda_{23}z} \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} \frac{(\lambda_{11}t)^n (\lambda_{21}x)^k}{n! k!} \sum_{m=0}^{\infty} \sum_{p=m}^{\infty} \frac{(\lambda_{13}t)^m (\lambda_{23}z)^p}{m! p!} \\ F_{t2}(x, z) = e^{-\lambda_{11}t - \lambda_{21}x} e^{-\lambda_{13}t - \lambda_{23}z} \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} \frac{(\lambda_{11}t)^{n+1} (\lambda_{21}x)^k}{(n+1)! k!} \sum_{m=0}^{\infty} \sum_{p=m}^{\infty} \frac{(\lambda_{13}t)^{m+1} (\lambda_{23}z)^p}{(m+1)! p!} \end{cases} \quad (24)$$

#### 4.2 Continuous Immersion Case

Mean Probability Density Function  $h_{tm}(x, z)$

$$h_{t1}(x, z) \leq \left\{ \left[ h_t(x, z) = \frac{1}{t_d} \int_0^{t_d} f_{t-\tau}(x) f_{t-\tau}(z) d\tau \right] \cong h_{tm}(x, z) \cong \frac{1}{2} [h_{t1}(x, z) + h_{t2}(x, z)] \right\} \leq h_{t2}(x, z) \quad (25)$$

$$\begin{cases} h_{t1}(x, z) = \frac{1}{t_d} \lambda_{21} \lambda_{23} \int_0^{t_d} e^{-\lambda_{21}x - \lambda_{23}z} e^{-(\lambda_{11} + \lambda_{13})(t-\tau)} \left\{ \sum_{n=0}^{\infty} \frac{[\lambda_{11}(t-\tau)]^n (\lambda_{21}x)^n}{n! n!} \right\} \left\{ \sum_{n=0}^{\infty} \frac{[\lambda_{13}(t-\tau)]^n (\lambda_{23}z)^n}{n! n!} \right\} d\tau \\ h_{t2}(x, z) = \frac{1}{t_d} \lambda_{21} \lambda_{23} \int_0^{t_d} e^{-\lambda_{21}x - \lambda_{23}z} e^{-(\lambda_{11} + \lambda_{13})(t-\tau)} \left\{ \sum_{n=0}^{\infty} \frac{[\lambda_{11}(t-\tau)]^{n+1} (\lambda_{21}x)^n}{(n+1)! n!} \right\} \left\{ \sum_{n=0}^{\infty} \frac{[\lambda_{13}(t-\tau)]^{n+1} (\lambda_{23}z)^n}{(n+1)! n!} \right\} d\tau \end{cases} \quad (26)$$

Mean Probability Distribution Function  $H_{tm}(x, z)$

$$H_{t1}(x, z) \leq \left\{ \left[ H_t(x, z) = \frac{1}{t_d} \int_0^{t_d} F_{t-\tau}(x) F_{t-\tau}(z) d\tau \right] \cong H_{tm}(x, z) \cong \frac{1}{2} [H_{t1}(x, z) + H_{t2}(x, z)] \right\} \leq H_{t2}(x, z) \quad (27)$$

The Distribution Functions  $F_{t-\tau}(x)$  and  $F_{t-\tau}(z)$  are obtained from the 1-D models' approximate equations in the longitudinal and vertical directions, respectively, that is, from the 1-D Homogeneous Poissonian equations systems with instantaneous immersion (Wilson-Jr and Monteiro, 2019).

### 5. SOME RESULTS WITH 2-D HOMOGENEOUS POISSONIAN MODELS

#### 5.1 The Mutually Independent 1-D Stochastic Process

The hypothesis that has allowed to successfully develop the 2-D Stochastic Models is the independence of the longitudinal and vertical movements in turbulent open flows. This property was described by Equation (18). It allows us to analyze the 1-D movements independently of each other and determining the values of the grain's mobility by simple equations, which may be adjusted to field surveys (Wilson-Jr and Monteiro, 2019).

#### 5.2 Lagrangean and Eulerian Results

Data obtained in a rectangular prismatic channel of the LCHF (Figure 5) have been used to validate and calibrate the 1-D and 2-D models. Figures 8 and 9 show results for uniformly distributed lateral injections on the free surface, and continuously during the time interval  $[0, t_d = 120 \text{ s}]$ .

The plumes in Figure 8 correspond to the 2-D movements of fine sediments depending on the diameter of the grain. The particles are supposed to have the same longitudinal mobility of the liquid and the vertical mobility proportional to their diameter. The figure clearly illustrates: the variation of vertical grain mobility according to their diameters, and how the particles are integrated into the flow and on the riverbed.

The plume in Figure 9 indicates the 2-D behavior of uniform sediments ( $D_{50} = 0.040 \text{ mm}$ ). The graphs represent: (i) the Lagrangean 2-D field of normalized concentration  $C(x, z, t)$ , in a channel 20.0 m long and 2.0 m deep, in the instant  $t = 100.0 \text{ s}$ ; (ii) Lagrangean 1-D vertical profiles of sediment concentration in sections  $x = 3.0; 10.0$  and  $17.0 \text{ m}$  in the instant  $t = 100.0 \text{ s}$ , and (iii) the Eulerian evolutions of the concentrations, in the levels  $z = 0.4$  and  $0.8 \text{ m}$ , in the three sections.

#### 5.3 The Bed Mobile Layer Roles

The fluid models the moving bed of the flow while it has its hydrodynamic characteristics modified by the forms that it has modeled itself. Thus, hydrodynamic and mass transfer studies must be carried out simultaneously. For solid particles of same physical and mineral properties, it is expected that the variables

that intervene in the bedload and suspended load movements are related to each other, at least in their common frontier: the mobile bed layer. Indeed, for the development, calibration and validation of 1-D and 2-D Stochastic Processes of sediment and pollutant which move as bedload and/or suspended-load with vertical transfers, there is a set of experimental results obtained in laboratory channels, creeks and rivers, using tracers where the mobile bed behaves in four distinct ways; as a(n):

- (i) **Supply source** of sediment grains which move in contact with the bed and/or in suspension, for example, as ripples and/or dunes.
- (ii) **Absorption barrier** of suspended sediments touching the mobile bed. Figure 10 presents the sediment deposits of a fine sand plume ( $D = 0.125$  mm), injected on the free surface, at a constant rate, during the interval  $[0, t_d = 200$  s] in flows of varying depths. The deposit at the bottom is a function of the diameter and of the specific weight of the grain, and of the hydrodynamic properties of the flow, which define the vertical  $\lambda_{1z}$  and  $\lambda_{2z}$ , and longitudinal  $\lambda_{1x}$  and  $\lambda_{2x}$  mobilities. Figure 11 illustrates the longitudinal distribution of sediment deposition of various diameters at the bottom of an open channel flow. The weighted composition of the final deposit of the material injected into the free surface was also presented. They correspond to the case of reflection 0.0 % in Figure 12.
- (iii) **Simultaneous barrier of absorption and reflection**. Figure 12 illustrates cases in which 100.0; 80.0; 60.0; 40.0; 20.0 and 0.00% of fine sediments ( $D = 0.0125$  mm) were absorbed by the riverbed.
- (iv) **Reflection barrier** of suspended sediments that touch the bed under the effect of turbulence and gravity, and return completely to the suspension, as the case of 100.0 % reflection in the Figure 12.

Grain Diameter D (mm)	Continuous Injection – Constant Longitudinal Mobilities $\lambda_{1x} = 2.97$ s <sup>-1</sup> ; $\lambda_{2x} = 10.00$ m <sup>2</sup> ; $U_x = 0.297$ m/s Immersion Period $t_i = 420.0$ s; Instantaneous Detection = 180.0 s	Vertical Mobility		
		$\lambda_{1z}$ s <sup>-1</sup>	$\lambda_{2z}$ m <sup>2</sup>	$U_z$ m/s
0.040 Coarse Silt		0.014	10.00	0.0014
0.080 Very Fine Sand		0.058	10.00	0.0058
0.100 Fine Sand		0.090	10.00	0.0090
0.125 Fine Sand		0.141	10.00	0.0141
0.160 Fine Sand		0.230	10.00	0.0230
0.200 Fine Sand		0.360	10.00	0.0360

Figure 8. Bidimensional movements of fine sediment injected continuously over a period, on the watercourse's free surface as a function of the grain diameter.  
(Wilson-Jr and Monteiro, 2019)

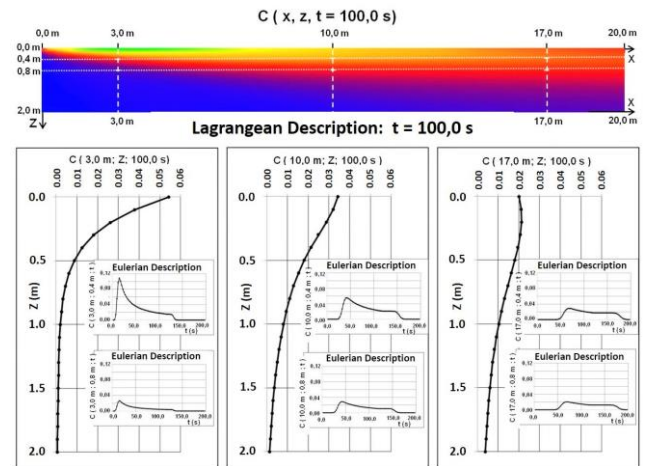


Figure 9. Lagrangean and Eulerian descriptions of uniform fine sediment's concentration  $C(x, z, t)$ .  
(Wilson-Jr and Monteiro, 2016, 2019).

## 6. CONCLUSIONS

The Theory of Stochastic Processes shows that the 2-D (longitudinal and vertical) trajectories of sediment and pollutant particles result from the combination of 1-D independent series of longitudinal and vertical displacements defined by the Mobility Intensity Functions:  $\lambda_{x1}$ ;  $\lambda_{z1}$ ;  $\lambda_{x2}$  and  $\lambda_{z2}$ , which analytical expressions describe the instantaneous and punctual movements of single particles.

This theory is so powerful that the classic Fickian diffusion and dispersion equations of suspended movements are only particular cases of Stochastic Processes, which are characterized by constant values of the Mobility Intensities, that is, by Homogeneous Poissonian Processes. However, when the mobilities are not constant, complex models must be used, such as the No-Homogeneous Poissonian, and No-Poissonian Models.

Results summarized in Figure 1 have been used for the development and understanding of the dynamics of the movement of sediments and pollutants in open-channel flows. The main phenomena to which the particles are subjected, such as turbulent diffusion, differentiated dispersion due to the gradients of speed field, sedimentation, solid material deposits and bed erosion, can be described with the help of Stochastic Processes.

For these investigations and applications, there is a collection of data on the movement of sediments and pollutants obtained in laboratory channels and nature, with the use of radioactive, dye and chemical tracers.

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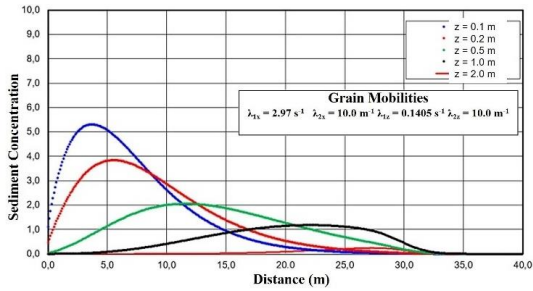


Figure 10. Sediment deposition of  $D = 0.125$  mm in flows of varying depths. Immersion Interval  $[0, t_d = 200$  s] (Wilson-Jr and Monteiro, 2019)

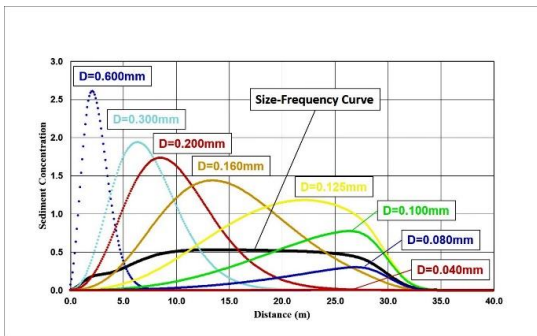


Figure 11. Longitudinal profiles of sediment deposits as a function of grain diameter.

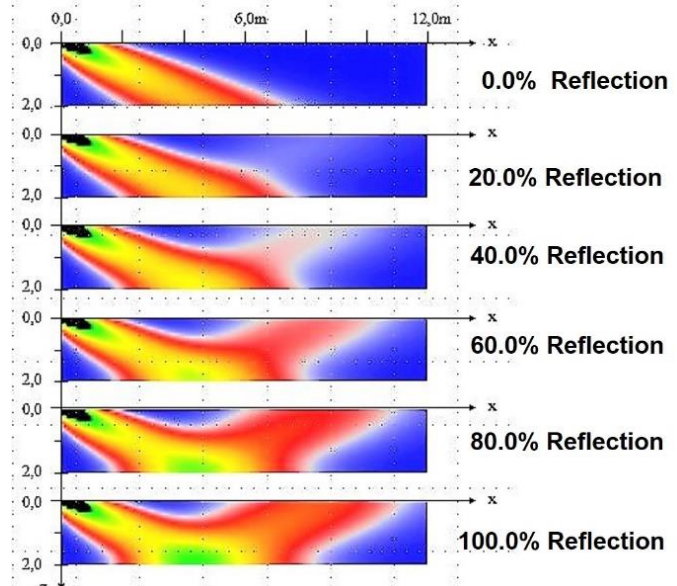


Figure 12. Mobile bed as a barrier of reflection (and/or absorption) of sediment grains ( $D = 0.0125$  mm) continuously injected on the free surface, during an interval  $[0, t_d]$ .

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