EROSION RATE OF BED SEDIMNET IN TERMS OF ENTRAINMENT CONCEPT

HARADA DAISUKE
International Centre for Water Hazard and Risk Management (ICHARM), Public Works Research Institute, Tsukuba, Japan,
e-mail :d-harada55@pwri.go.jp

EGASHIRA SHINJI
International Centre for Water Hazard and Risk Management (ICHARM), Public Works Research Institute, Tsukuba, Japan,
e-mail :s-egashira77@pwri.go.jp

ABSTRACT

In the estuary bed composed of very fine sediment material, tidal currents cause active sediment transportation, bar morphology changes and severe bank erosion. However, it is difficult to evaluate the erosion rate of bed sediment using methodologies developed for suspended sediment materials in previous researches. The present study proposes a new method to evaluate the erosion rate of sediment bed composed of silt-clay material by introducing the concept of entrainment for evaluating a mixing process of density stratified flow.

Nineteen cases of flume experiments are conducted to investigate the erosion rates of bed material with different concentrations of water-sediment mixture, sediment particle sizes and Richardson numbers. The results of the flume experiments are then compared with the results of the experiments conducted for density stratified flows. The results are summarized in terms of the entrainment coefficient and the Richardson number. The results show that the erosion rate of bed sediment can be evaluated using a modified Richardson number which is introduced to evaluate the influence of viscosity on entrainment rates.

Keywords: Very fine sediment, Erosion rate, Entrainment velocity, Density current

1. INTRODUCTION

The river bed of an estuary or a dam reservoir is often composed of very fine sediment particles measuring tens of micrometers. Such fine materials are easily eroded and transported by the flow, and these phenomena cause morphological changes. For example, in the Sittaung River estuary in Myanmar, bed materials are composed of particles measuring a mean diameter of around 30 micrometers; hence, active sediment transport occurs, resulting in severe morphological changes, including active bank erosion (Ahmad et al., 2018).

In order to evaluate such erosion processes of fine sediment, the idea of the erosion rate of suspended sediment has been studied. In particular, methods to evaluate reference concentration have been proposed in many studies (Lane and Kalinske, 1941; Itakura et al., 1981). However, in the case of a riverbed composed of silt-cray particles with their medium sizes ranging 20 to 40 micrometers, the erosion rate, when evaluated using those methods, shows an unrealistically large value due to slow settlement velocities of such materials. An alternative method needs to be developed for more accurate evaluation of the erosion rate. Although a lot of studies have tried to treat bed deformation in the flow fields where suspended sediment dominate (e.g., Yu et al., 2012, Chou et al., 2018), it is not easy to find studies on erosion processes such as present topics.

The concept of entrainment, which employs the methodology of density stratified flows, can be a powerful tool for this purpose. This concept theorizes that the difference of the density between two layers causes entrainment of the material from one layer to the other, and studies have revealed that the entrainment coefficient is inversely proportional to the Richardson number (Ellison and Turner, 1959; Ashida and Egashira, 1976). This study aims to develop a method to evaluate the erosion rate of bed sediment composed of very fine materials by employing the concept of entrainment and also to investigate the validity of the method by conducting flume tests.

2. BEDLOAD AND SUSPENDED LOAD OF VERY FINE SEDIMENT

2.1 Bedload

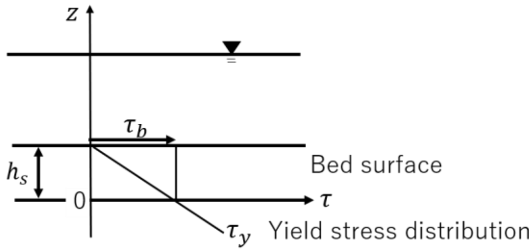

Figure 1. Yield stress distribution in the bed surface layer

Figure 1 shows a schematic image of vertical stress distribution in the bed surface layer. The bed sediment is composed of very fine materials, which have loosely deposited. The shear stress, τ_b, affects the bed surface. The bed sediment has cohesion since the bed is composed of very fine materials, and thus the cohesion increases in the deep layer direction. Based on the assumption that the vertical cohesion distribution is proportional to the normal stress, the vertical distribution of yield stress is explained as follows by introducing the apparent internal friction angle:

$$\tau_y = \rho(\sigma/\rho - 1)c_s g(h_s - z)\tan\phi \tag{1}$$

where τ_y is the vertical distribution of yield stress, ρ is the water density, σ is the density of sediment particle, c_s is the concentration of sediment in the bedload layer, and g is the gravitational acceleration.

According to Figure 1, shear stress τ_b is equal to τ_y at $z = 0$. Thus, the thickness of the bedload layer is described as follows:

$$\frac{h_s}{h} = \frac{i_e}{(\sigma/\rho - 1)c_s\tan\phi} \tag{2}$$

where h is the water depth, $\tau_b = \rho g h i_e$, and i_e is the energy slope.

In the case that the bedload layer ($0 < z < h_s$) is the laminar flow:

$$\rho\nu\frac{\partial u_s}{\partial z} = \tau_b - \tau_y \tag{3}$$

where ν and u_s are the kinematic viscosity coefficient and the velocity in the bedload layer ($0 < z < h_s$), respectively.

Equation (3) is transformed to:

$$\frac{\partial u_s}{\partial z} = \frac{{u_*}^2}{\nu}\left\{1 - \frac{(\frac{\sigma}{\rho} - 1)c_s}{i_e}\frac{h_s}{h}\left(1 - \frac{z}{h_s}\right)\tan\phi\right\} \tag{4}$$

Therefore, the following equation is obtained by substituting equation (2) into equation (4):

$$\frac{\partial u_s}{\partial z} = \frac{u_*^2}{\nu}\frac{z}{h_s} \tag{5}$$

Vertical velocity distribution $u_s(z)$ and mean velocity V are therefore defined as follows:

$$u_s(z) = \frac{1}{2}\frac{u_*^2}{\nu}\frac{z^2}{h_s} \tag{6}$$

$$V = \frac{1}{6}\frac{u_*^2}{\nu}h_s \tag{7}$$

Bedload transport rate q_b is as follows using the relationships of $q_b = c_s V h_s$:

$$q_b = \frac{c_s}{6}\frac{u_* h_s}{\nu}u_* h_s \tag{8}$$

2.2 Entrainment of bed surface material

Figure 2 shows the mass conservation of very fine sediment in the water in a one-dimensional field. The bed sediment is entrained into the water at entrainment velocity W_e, and the fine sediment in the water deposits at settling velocity w_0. Therefore, the mass conservation of the fine sediment in the water is described as the following advection-dispersion equation:

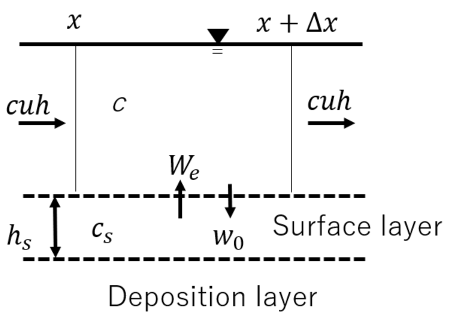

Figure 2. Mass conservation of very fine sediment

$$\frac{\partial ch}{\partial t} + \frac{\partial cuh}{\partial x} = \frac{\partial}{\partial x}\left(\varepsilon h \frac{\partial c}{\partial x}\right) + W_e c_s - w_0 c \tag{9}$$

where c is the concentration of fine sediment particle in the water, u is the mean velocity, and ε is the dispersion coefficient.

The continuity equation of water is described as follows:

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} = W_e - w_0 \frac{c}{c_s} \tag{10}$$

From equations (9) and (10), the suspended sediment concentration is described as follows:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = \frac{1}{h}\frac{\partial}{\partial x}\left(\varepsilon h \frac{\partial c}{\partial x}\right) + \frac{1}{h}\left\{W_e c_s(1 - \frac{c}{c_s}) - w_0 c(1 - \frac{c}{c_s})\right\} \tag{11}$$

In the case of the equilibrium condition, in which the sediment entrainment into the water and the sediment deposition to the bed are in balance, equilibrium concentration c_e is described as follows:

$$c_e = \frac{W_e}{w_0} c_s \tag{12}$$

By introducing entrainment velocity W_e used in past studies such as the Ellison and Turner (1959) and Ashida and Egashira (1976), the entrainment velocity is given as follows:

$$\frac{W_e}{u}(= e) = \frac{K}{R_{i*}} \qquad (R_{i*} = \frac{\Delta\rho}{\rho} gh/u^2) \tag{13}$$

where R_{i*} is the Richardson number, e is the entrainment coefficient, and $\Delta\rho$ is the difference in the density between the two layers. According to Ashida and Egashira (1976), $K = 1.5 \times 10^{-3}$.

From equations (12) and (13), equation (14) is defined as:

$$c_e = \frac{1}{\frac{w_0}{u_*}} \sqrt{\frac{8}{f}} K \frac{\rho}{\Delta\rho} \frac{u^2}{gh} \tag{14}$$

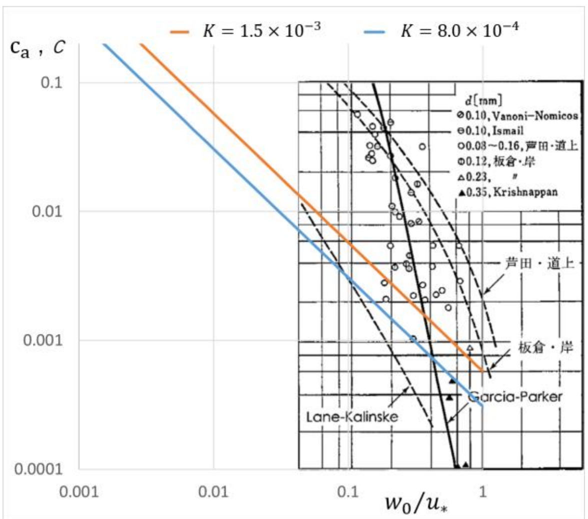

Figure 3. Comparison of the suspended sediment concentration between the methods proposed on the present study and the results proposed in the previous studies.

where, f is the friction loss coefficient. The relationship of $u/u_* = \sqrt{8/f}$ is employed to derive equation (14).

Figure 3 compares the current methods proposed on equations (11) to (14) with the results obtained by previous studies. In Figure 3, the parameters in equation (14) is specified as: F_r=0.1, f=0.05, c_s=0.2, $\Delta\rho/\rho$=0.33. The horizontal axis, w_0/u_*, extends to the order of 10^{-2} or 10^{-3}, which is the focus of the present study.
According to Figure 3, in the area where w_0/u_* is small, which the present study focuses, the sediment concentration proposed on this study shows realistic values compared to the equations suggested in the previous studies. This result implies the applicability of the proposed method.

3. FLUME EXPERIMENT

A series of flume experiments are conducted to check the applicability of the method to proposed in the present study, using 'pearl- cray', known a composed of very fine particles. Figure 4 shows the particle size distribution as a result from sedimentation particle size analysis. The mean diameter of the particles is approximately 5μm.

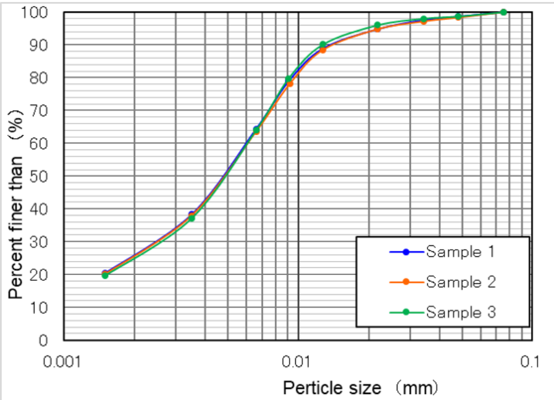

Figure 5. Particle size distribution of the material employed for the experiments.

A flume, 11m long and 0.2m wide, is prepared as shown in Figure 6, and the maximum discharge is set at 20*l/s*. A space is created around the center of the flume to store a mixture of water and pearl- cray. Since the measured sediment concentration is larger as the length of the space is longer, the space is designed to change its length from 1m to 3m to check the influence of the length on the entrainment velocity.

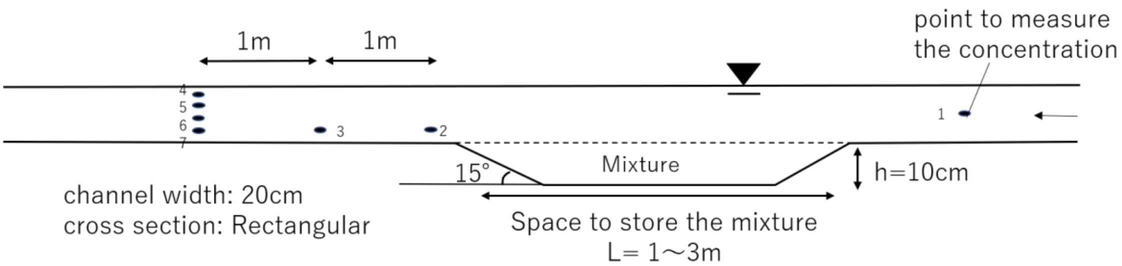

Figure 6. Schematic image of the flume employed for the experiment.

After the mixture of water and pearl- cray is set in the space, the flume is filled with water carefully so that sediment particles would not be entrained into the water at that moment (See Figure 7). After the flume is filled with water, the flow starts. During the experimental periods, when the erosion rate seems constant, the sediment concentration is measured at the 7 points shown in Figure 6. The water is collected for 10 seconds at each point. The collected samples are then dried to calculate the sediment concentration. Measurement points 4 to 7 are set at different depths of 20%, 40%, 60% and 80%, respectively. The results of the flume experiments in Figure 7 are the mean values of these measurement points.

Table 1 shows the experimental cases in this study. The height of the mixture is set slightly lower than the height of the flume, since the downstream edge of the mixture is lifted up due to the flow.

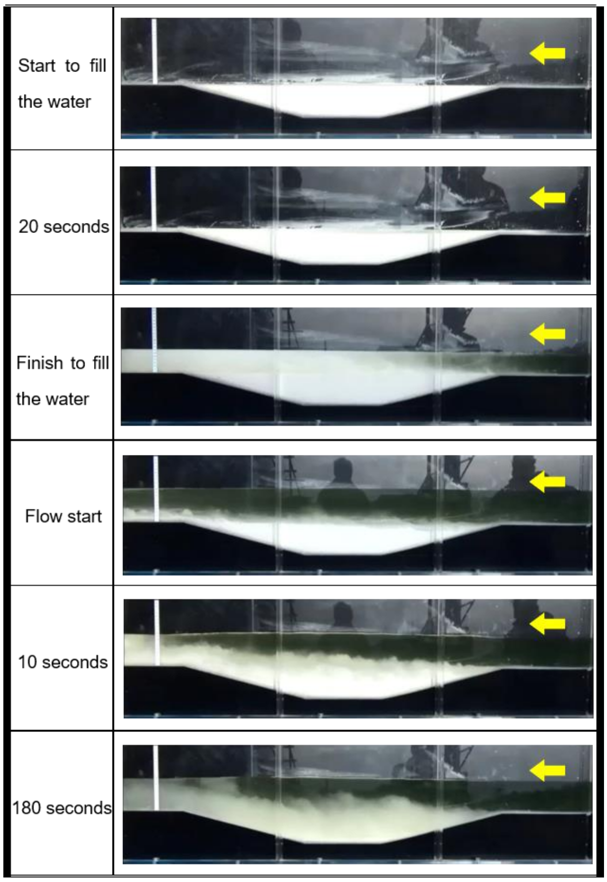

Figure 7. Longitudinal photos of a flume experiment

Table 1. Cases of the experiments and the results

CASE	L (m)	Discharge (L/s)	Water depth (cm)	Storage depth (cm)	Concentration (%)	We/u	Ri*	$\frac{\nu_c}{\nu_0}$
1-1-1	1.0	3.5	8.0	9.0	12.0	6.6E-06	6.34	7.5
1-1-2					17.0	3.6E-06	8.81	18.0
1-1-3					22.0	1.4E-06	11.56	55.0
1-2-1		7.0	8.0	10.0	12.0	2.3E-03	2.97	7.5
1-2-2					17.0	1.1E-03	2.65	18.0
1-2-3					22.0	1.1E-04	2.28	55.0
1-3		7.0	8.0	7.0	12.0	2.1E-04	3.37	7.5
1-4		5.0	7.0	8.0	8.0	1.4E-03	5.35	4.5
1-5-1		10.0	10.0	7.0	8.0	2.1E-03	2.53	4.5
1-5-2					12.0	7.2E-04	3.08	7.5
1-5-3					17.0	4.3E-05	3.53	18.0
1-5-4					22.0	5.5E-06	4.16	55.0
3-1-1	3.0	5.0	7.0	8.0	8.0	4.1E-04	3.73	4.5
3-1-2					12.0	1.9E-04	4.62	7.5
3-1-3					17.0	6.4E-06	4.24	18.0
3-1-4					22.0	1.1E-06	4.92	55.0
3-2-1		10.0	10.0	7.0	8.0	9.9E-04	2.53	4.5
3-2-2					12.0	6.1E-04	3.37	7.5
3-2-3					17.0	5.5E-05	3.75	18.0
3-2-4					22.0	4.9E-06	4.69	55.0
3-3		5.0	10.0	8.0	8.0	1.4E-04	6.52	4.5

4. RESULTS AND DISCUSSIONS

During the experiments, internal waves are formed at the boundary of the mixture and the water with a wave length of several millimeters to several centimeters, causing sediment erosion. A similar mechanism of sediment entrainment has been reported by some previous studies such as Sekine et al. (2003).

The experimental results are shown in Figure 8 (left). The entrainment velocities and the entrainment coefficients are computed using the following equation derived from equation (9):

$$\frac{\partial cuh}{\partial x} = W_e c_s \quad (15)$$

In Figure 8 (left), the entrainment coefficients obtained by the experiments are compared with the results of the previous studies obtained from the investigation of the density stratified flow. According to Figure 8 (left), for the cases of a low concentration of the mixture, i.e., when the cohesion is not so dominant, the erosion rate is properly evaluated by the entrainment coefficients. On the other hand, especially in the cases that the concentration of the mixture is higher than 17%, the entrainment velocity is significantly small compared to those of the previous studies. This difference is assumed to be caused by the difference in cohesion, which affects the entrainment velocity more strongly when the concentration of the mixture is high.

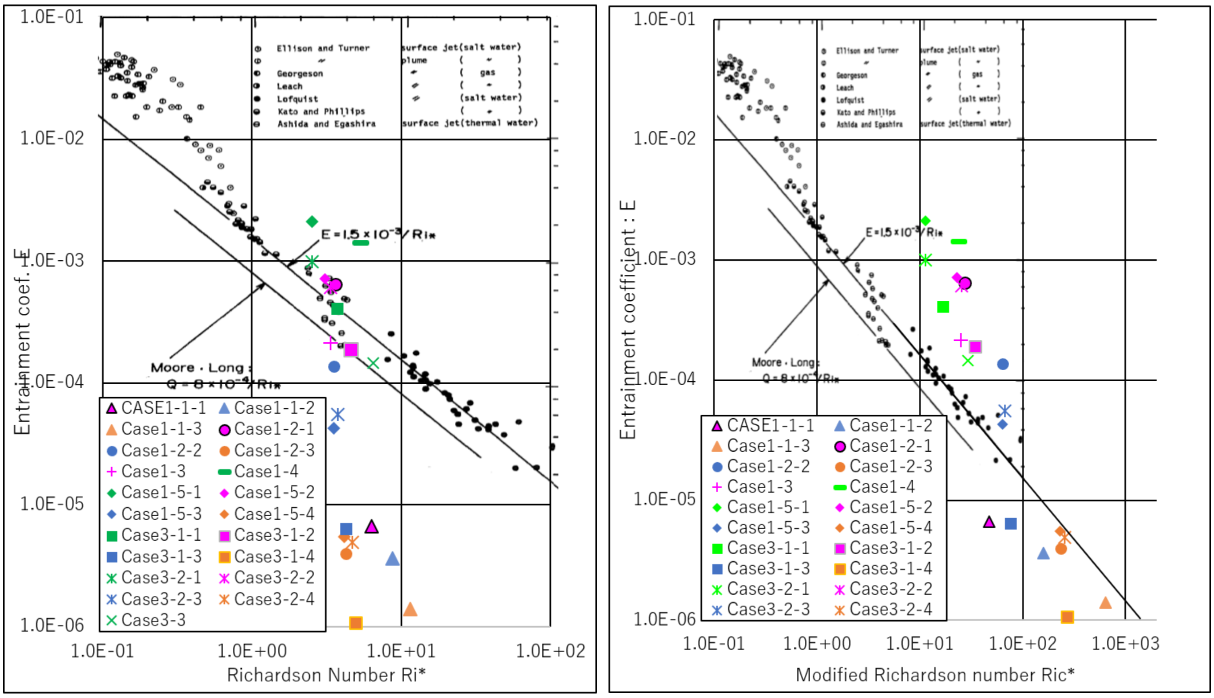

Figure 8. Comparison of the experimental results and the entrainment coefficients proposed by previous studies: Original (left) and Modified Richardson number (Right)

There may be several methods for evaluating the influence of cohesion on the entrainment coefficient. It is considered that the shear stress acting on the interface between the water-sediment mixture and the upper layer is a key parameter and a part of work rate due to the shear stress causes the entrainment:

$$\tau u \sim \Delta\rho g h W_e \quad (16)$$

Equation (16) is reformed:

$$\frac{W_e}{u} \sim \frac{\tau}{\Delta\rho g h} \quad (17)$$

In equation (17), it is considered that the shear stress at interface must be laminar flow because flux Richardson Number is large so that the turbulent motion decays. Therefore, the following equation is derived:

$$\frac{W_e}{u} \sim \frac{\nu \frac{\partial u}{\partial z}}{u^2} \frac{1}{R_{i*}} \quad (18)$$

Assuming that the entrainment coefficient of water-sediment mixture can be evaluated by equation (18), we obtain:

$$\frac{\nu_0 \frac{\partial u}{\partial z}}{u^2} \frac{1}{R_{i*0}} = \frac{\nu_c \frac{\partial u}{\partial z}}{u^2} \frac{1}{R_{i*c}} \quad (19)$$

where ν_0 and R_{i*0} are the kinematic viscosity and the Richardson number of the water, and ν_c and R_{i*c} is the kinematic viscosity and the Richardson number of the cohesive material. Therefore, the following relation is obtained:

$$R_{i*c} = \frac{\nu_c}{\nu_0} R_{i*0} \quad (20)$$

Based on equation (20), R_{i*} in Figure 8 (Right) shows the relation between the entrainment coefficient and the modified Richardson number, which is modified using the values shown in Table 2. In Table 2, the values of ν_c/ν_0 are specified from Ashida et al. (1986). According to Figure 8 (right), the entrainment coefficient can be explained using the modified Richardson number. Therefore, the erosion rate of bed sediment composed of very fine material is evaluated by the present method, in which the modified Richardson number shown in Equation (20) is applicable for the cohesive material.

5. CONCLUSIONS

This study proposed a method to evaluate the erosion rate of very fine sediment particles by introducing the concept of entrainment. The convection-diffusion equation for the fine sediment transportation is proposed using an entrainment concept, thus an equation to evaluate an erosion rate of bed sediment is proposed. In order to test the validity of the method, a series of experiments are conducted. The experiment results show that the erosion rate can be evaluated properly using the entrainment coefficient proposed on the previous studies at least for the cases with a low concentration of the mixture, in which the cohesion is not so dominant. On the other hand, though the entrainment velocity is small, especially in the cases in which the concentration of the mixture is higher than 17% due to a strong cohesion, the erosion rate can be evaluated using a modified Richardson number which is introduced to evaluate the influence of viscosity on entrainment rates.

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