VARIABILITY IN STAGE-DISCHARGE RELATIONSHIPS IN RIVER REACH WITH BED EVOLUTIONS

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ABSTRACT

Stage-discharge relationships reflect the hydraulic behavior of a river reach. In the present study, the authors aim to investigate the dynamic relationships between stage and discharge numerically in a river reach where riverbed evolutions take place actively and to propose a simple method for predicting the water levels during flood. The numerical simulations are conducted by employing a one-dimensional numerical model for experimental channels assuming rigid and erodible riverbed as well as for Seri River, Japan. Results obtained from the numerical computations show that the behavior of the stage-discharge relation for flows over erodible bed differs significantly from those for flow over rigid bed in the reaches where flow width is not constant. The computed results suggest that the one-dimensional method is useful for a river reach having large curvature radius.

Keywords: Riverbed evolution, numerical modelling, stage-discharge relationship, channel geometry

1. INTRODUCTION

Water levels result from complex interactions among many variables and are important for flood management. Water levels are almost exclusively evaluated based on in-situ observations or employing empirically specified relation between water levels and flow discharges known as rating curve. The assumption of steady and uniform flow in constructing stage-discharge relationship curves are common which provide unique and monotonically increasing correlation. Such curves are relatively less complicated to formulate and several methods such as graphical analysis, non-linear regression analysis and data assimilation techniques are available in numerous literatures (Kennedy, 1984; Schmidt and Yen, 2001; Petersen-Øverleir, 2004; Moyeed and Clarke, 2005; Herschy, 2009; Mansanarez *et al.*, 2016). Generally, streamlines maintain its uniformity in the flow direction in the case of nearly constant geometry. However, mountainous rivers exhibit heterogeneous channel geometry in addition to the unsteady flow and prevail with riverbed evolutions. As such, a unique correlation may not be sufficient to express water level explicitly for all phases of flood propagation.

The non-uniqueness in stage-discharge relation resulting from flow unsteadiness is termed as hysteresis or loop and has been studied by many researchers from the hydrodynamics perspective. The physical background of the hysteresis behavior together with in-depth discussions on governing equations associated with unsteady flow are described in many literatures and text books (such as, Jones, 1916; Lighthill and Whitham, 1955; Henderson, 1966; Fread, 1975; Perumal and Ranga Raju, 1999; Perumal *et al.*, 2004). Several methods and formulas are also available in several publications and text books to adjust the steady state stage-discharge relationship curves for the unsteady flow. A comprehensive review of such existing methods are included in Dottori *et al.* (2009). Among them, the simplest approach is to develop separate curves for rising and falling phase of the flood by means of data fitting techniques based on the measured discharges. Nevertheless, it is challenging to conduct continuous discharge measurement in the natural rivers and behavior of the hysteresis curves differ considerably as well in each flood events (Mishra and Seth, 1996; Graf and Qu, 2004). Moreover, morphodynamic activities of river channels and anthropogenic endeavors which include riverbed variations, volume of incoming sediment and its grain size distributions, grain-size distributions of riverbed materials, amounts of channel or floodplain storage, backwater effects, etc. influence the water surface profile substantially.

Spatial and temporal changes in channel geometry due to riverbed evolutions introduce added complexities in the stage-discharge relationship besides the flow unsteadiness (Simons and Richardson, 1962). Present study aims to discuss dynamic relationships between water levels and discharge numerically in a river reach where riverbed evolutions take place actively and to propose a very simple method for evaluating riverbed evolutions and water levels during flood. Numerical simulations are conducted by employing a one-dimensional governing equations for water flow and sediment transport in experimental channels assuming rigid and mobile riverbed as well as in Seri River, Japan. Present study employs a bedload layer thickness which varies corresponding to the non-dimensional bed shear stress instead of constant exchange layer thickness in conjunction with a bedload formula having its functional form of $\tau_{*i}^{2.5}$ (Egashira *et al.*, 1997) in order to deal with issues associated to non-uniform sediment transportation.

2. GOVERNING EQUATIONS

2.1 1-D Model for flow and riverbed evolution

Considering an erodible wide rectangular channel carrying flow discharge of $Q(x, t)$ having cross-sectional area of $A(x, t)$ with a flow depth of $h(x, t)$, flow width of $B(x, t)$ and mean elevation of riverbed $\eta(x, t)$ at the longitudinal coordinate x and time t as shown in Figure 1 and assuming no lateral inputs are presented, the 1-D governing equations for flow field are written in the following form:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial Y}{\partial x} + g\frac{n^2 Q^2}{Ah^{4/3}} = 0 \tag{2}$$

where, $Y(x, t) = h(x, t) + \eta(x, t)$ is the free surface elevation, n is the Manning's roughness coefficient, g is the acceleration due to gravity. In order to complete the morphological evolution of the riverbed, the 1-D form of the mass conservation of riverbed sediment is described as follows.

$$(1-\lambda)\frac{\partial \eta}{\partial t} + \sum_i \left(\frac{1}{B}\frac{\partial B q_{bi}}{\partial x} + E_i - D_i + E_{wi} - D_{wi}\right) = 0 \tag{3}$$

where, λ is the porosity, q_{bi} is the bedload transport rate for sediment size class-i in the bedload layer, E_i and D_i are the erosion and deposition rate of suspended sediment for sediment size class-i, E_{wi} and D_{wi} are the erosion and deposition rate of wash load for sediment size class-i. The dimensionless bedload transport rate is evaluated by Egashira *et al.*'s formula as:

$$q_{b*i} = p_i \frac{4}{15}\frac{K_1^2 K_2}{\sqrt{f_f + f_d}}\tau_{*i}^{2.5} \tag{4}$$

where, q_{b*i} is the dimensionless sediment transport rate for sediment size class-i; τ_{*i} is the non-dimensional bed shear stress for sediment size class-i, p_i is the fraction of sediment size class-i. Parameters K_1, K_2, f_f, f_d and non-dimensional bed-shear stress are defined as:

$$K_1 = \frac{1}{\cos\theta}\frac{1}{\tan\varphi - \tan\theta}, \quad K_2 = \frac{1}{c_b}\sqrt{1 - \frac{h_s}{h}} \tag{5}$$

$$\frac{h_s}{h} = \frac{1}{\left(\frac{\sigma}{\rho} - 1\right)c_b}\frac{\tan\theta}{\tan\varphi - \tan\theta} \tag{6}$$

$$f_d = k_d(1 - e^2)\left(\frac{\sigma}{\rho}\right)c_b^{1/3}, \quad f_f = k_f(1 - c_b)^{5/3}c_b^{-2/3} \tag{7}$$

$$\tau_{*i} = \frac{u_*^2}{\left(\frac{\sigma}{\rho} - 1\right)g d_i} \tag{8}$$

where, e is the restitution coefficient, u_* is the friction velocity, h_s is the bedload layer thickness, k_d =0.0828, k_f =0.16, σ is the mass density of bed sediment, ρ is the mass density of water, θ is the local energy slope, φ is the angle of friction of riverbed sediment, c_b is approximated as $c_b = c_*/2$ is the average sediment concentration within bedload layer, c_* is the sediment concentration at stationary layer. The bedload transport rate (q_{bi}) per unit width takes following form:

$$q_{bi} = p_i \frac{4}{15}\frac{K_1^2 K_2}{\sqrt{f_f + f_d}}\frac{u_*^5}{\left(\frac{\sigma}{\rho} - 1\right)^2 g^2}\frac{1}{d_i} \tag{9}$$

Eq. (9) indicates that sediment transport rate is inversely proportional to the diameter of bed sediment. Therefore, larger sediment particle's transport rate is smaller than that of the finer one.

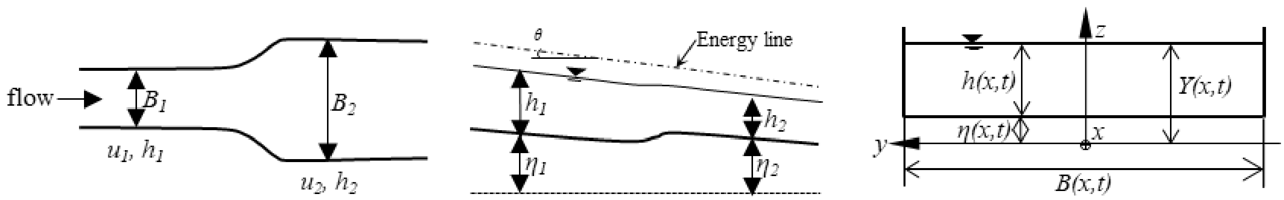

Figure 1. Schematic sketch for erodible channel with notation of symbols and water surface profile.

2.2 Bedload layer model

Present study employs bedload layer model to deal with issues associated to non-uniform sediment transport. In the model, the riverbed is divided into a bedload layer, a transition layer and numbers of vertical layers. Details of the model are included in Takebayashi (2005), Harada et al. (2018) and Biswas et al. (2019). The bedload layer thickness is estimated as a function of non-dimensional bed shear stress (Egashira *et al.*, 1997):

$$\frac{h_s}{d} = \frac{1}{c_b \cos\theta (\tan\varphi - \tan\theta)} \tau_* \tag{10}$$

where, d is the sediment size in the bedload layer. Selective transportation of bed sediment takes place due to the differences in sediment particle's mobility and are responsible for sediment sorting and riverbed armoring. The critical bed shear stress is influenced by relative size of sediment particle under consideration with respect to mean diameter of non-uniform sediment. Modified Egiazaroff equation and Iwagaki's formula is commonly applied in Japan for evaluating the critical bed shear stress for each grain-size of non-uniform sediment. In the present study, maximum size of sediment particle which can be transported by fluid force is obtained from Eq. (17), based on criteria introduced by Harada et al. (2018).

$$d_{cr} = c_b \cos\theta (\tan\varphi - \tan\theta) \frac{h_s}{\tau_{*c}} \tag{11}$$

where, τ_{*c} is the critical non-dimensional bed shear stress having a value of 0.05 and d_{cr} is the critical diameter of sediment particle for movement.

3. MORPHODYNAMIC EQUILIBRIUM OF NON-UNIFORM CHANNEL

An erodible channel exhibits morphodynamic equilibrium when its boundaries do not undergo any changes over time: hence, $\partial\eta/\partial t \approx 0$ and $\partial B/\partial t \approx 0$. Non-uniform river channel with fixed bank respond to channel widening and narrowing by adjusting the flow depth, sediment size distributions of riverbed materials (Parker, 1990) and the river planform in order to transport the incoming sediment without aggradation and degradation. Under such constraints, the continuity equation of water flow and bedload, and the energy conservation are reduced to the following forms.

$$B_1 h_1 u_1 = B_2 h_2 u_2 \tag{12}$$

$$B_1 q_{b1} = B_2 q_{b2} \tag{13}$$

$$\eta_1 + h_1 + \frac{u_1^2}{2g} = \eta_2 + h_2 + \frac{u_2^2}{2g} - i_e \Delta x \tag{14}$$

where, B is the flow width, q_b is the bedload transport rate, i_e is the energy slope. Other symbols are defined in Figure 1.

In an equilibrium stage of sediment transportation where all grains are transported actively, the flow resistance can be evaluated by means of Manning's formula;

$$u = \frac{1}{n} i_e^{\frac{1}{2}} h^{\frac{2}{3}} \tag{15}$$

A general functional form of the bedload transport rate formula for high bed shear stress is described as;

$$q_b \sim \sqrt{\left(\frac{\sigma}{\rho} - \right) g d^3}\, \tau_*^m \sim \sqrt{\left(\frac{\sigma}{\rho} - \right) g d^3} (h i_e)^m \tag{16}$$

where m is the exponent of non-dimensional bed shear stress. Using formulas (12) to (16), we obtain following regime relationship.

$$\frac{h_2}{h_1} = \left(\frac{B_2}{B_1}\right)^{\frac{3(1-2m)}{7m}} \left(\frac{n_2}{n_1}\right)^{6/7} \left(\frac{d_2}{d_1}\right)^{\frac{9}{14m}} \tag{17}$$

$$\frac{i_{e2}}{i_{e1}} = \left(\frac{B_2}{B_1}\right)^{\frac{2(3m-5)}{7m}} \left(\frac{n_2}{n_1}\right)^{-6/7} \left(\frac{d_2}{d_1}\right)^{\frac{-15}{7m}} \tag{18}$$

$$\frac{Fr_2^2}{Fr_1^2} = \left(\frac{B_2}{B_1}\right)^{\frac{4m-9}{7m}} \left(\frac{n_2}{n_1}\right)^{-18/7} \left(\frac{d_2}{d_1}\right)^{\frac{-9}{14m}} \tag{19}$$

$$\frac{\eta_2 - \eta_1}{h_1} = 1 - \left(\frac{B_2}{B_1}\right)^{\frac{3(1-2m)}{7m}} \left(\frac{n_2}{n_1}\right)^{6/7} + \frac{1}{2} Fr_1^2 \left[1 - \left(\frac{n_2}{n_1}\right)^{-12/7} \left(\frac{B_2}{B_1}\right)^{-\frac{2m+6}{7m}} \left(\frac{d_2}{d_1}\right)^{\frac{-9}{7m}}\right] - \frac{i_e \Delta x}{h_1} \tag{20}$$

In case of very high bed shear stress when all the particles of riverbed materials are transported actively d_1=d_2 and therefore n_1=n_2. Under such circumstance, the relationship obtained from Eqs. (17), (18), (19) and (20) are shown in Figure 2 where 2(a), (b), (c) and (d) are the profile of equilibrium values of ratios of the flow depths, riverbed evolutions, energy slopes, and Froude numbers, respectively for *m*=3/2 (Ashida and Michiue, 1972) and 5/2 (Egashira *et al*., 1997). The profiles of depths exhibit an increasing and decreasing trend corresponding to the channel narrowing and widening respectively in both cases. The profile of Froude numbers obtained based on *m*=5/2, as shown in Figure 2(d), suggests that the flow regime does not change largely for channel narrowing or widening. Figure 2(c) exhibits that the energy slope at downstream are steeper and milder for channel narrowing and widening, respectively for *m*=5/2 comparing to the upstream energy slope. However, trend differs considerably from the former one for *m*=3/2. Riverbed evolution takes place at the channel narrowing and widening as indicated in Figure 2(b). The riverbed degradation at channel narrowing is more sensitive to the change in channel geometry than that of riverbed aggradation process. The profiles also exhibit that evaluation of the riverbed evolution as well as flow depth is influenced by the choice of the functional form of the sediment transport rate.

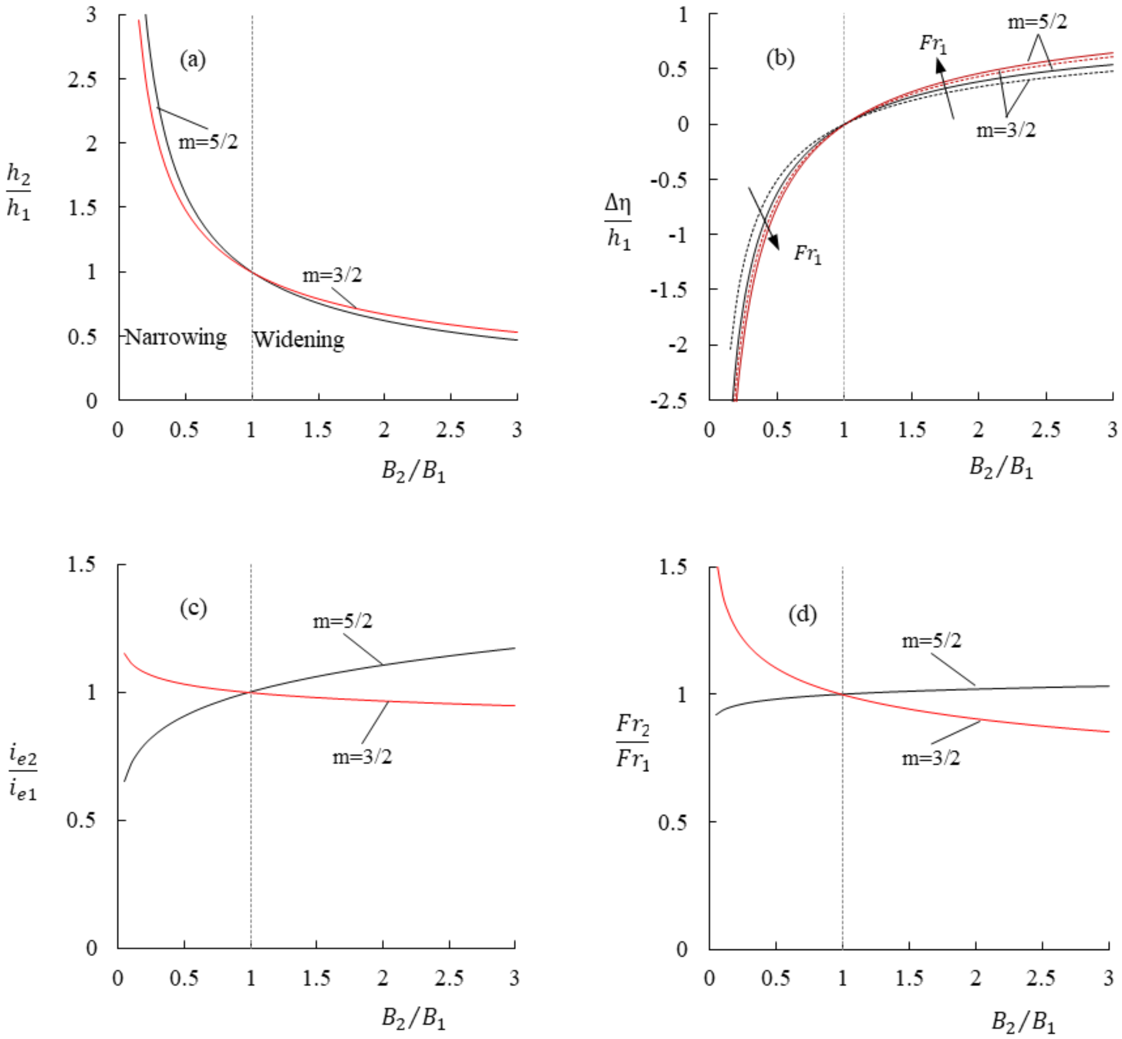

Figure 2. Equilibrium values of ratios of (a) flow depths, (b) Froude numbers, (c) friction slopes and (d) riverbed changes as a function of width ratios based on Eqs. (17) ~ (20) for exponent of non-dimensional bed shear stress *m*=5/2 and 3/2, considering very high bed shear stress with d_1=d_2 and n_1=n_2.

4. RESULTS AND DISCUSSION

4.1 Calculation conditions

The experimental channel having flatbed initially is employed for the numerical simulations. The numerical simulations are conducted for constant discharge as well as for the time series of flow discharges. Table 1 summarizes the calculation conditions and configurations of experimental channels for numerical simulations. The value of manning's roughness is set as 0.03 which is constant for entire simulations. The effect of riverbed steepness on the riverbed evolution and flow depth in the case of variable channel geometry is evaluated by numerical simulations with experimental channels having riverbed slope 1 in 1000 and 1 in 200, respectively for a constant discharge of 150 m^3/s and keeping other factors constant. The time series of flow discharge which have been supplied at the upstream boundary are shown in Figure 3(a) in order to evaluate the variations in water levels and discharge relationship under active riverbed evolution with experimental channel besides Seri

River, Japan. The time series is constructed artificially using flood event 2013 of Seri River, Japan in addition to flood events from 2014 to present. The riverbed material is composed of non-cohesive, non-uniform sediment having mean diameter of 18.2 mm with a standard deviation of 4.70 are shown in Figure 3(b). The minimum and maximum diameters of riverbed material are 0.09mm and 100mm, respectively. Transportations of bedload, suspended load and wash load have been taken into consideration for computing the riverbed variations. The bedload is supplied at a constant rate at the upstream boundary. The erosion rate terms for suspended sediment are evaluated using fall velocity of sediment particles and suspended sediment concentration at reference level. The equilibrium concentration at the reference level and particle fall velocity of suspended sediment specified using formulas proposed by Lane and Kalinske (1941) and Rubey (1933), respectively. The erosion rate term for wash load is appraised based on riverbed degradation as a proportion to the fraction of wash load component whereas it is considered to be zero for riverbed aggradation. The initial suspended sediment concentration is defined assuming an equilibrium suspended sediment concentration profile.

Table 1. Calculation condition with straight rectangular experimental channel.

CHANNEL TYPE	B_2/B_1	Q (m³/s)	i_b	i_b
Expansion	1.5 1.4 1.2	150	0.005	0.001
Contraction	0.6 0.7 0.8			

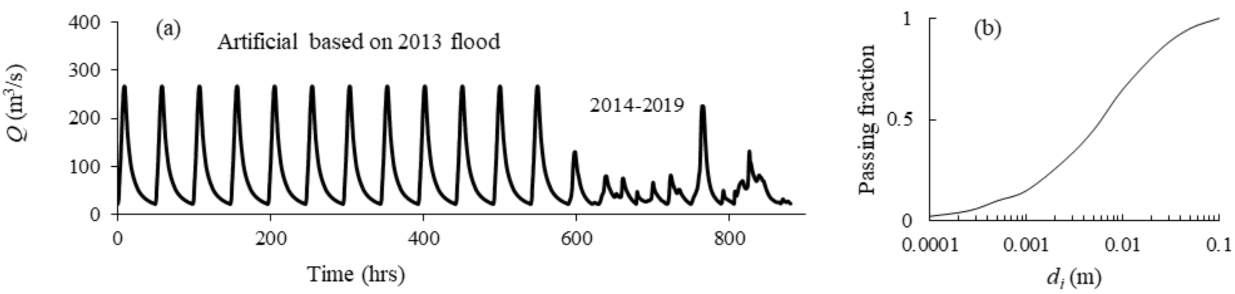

Figure 3. Calculation condition: (a) time series of flow discharge supplied at upstream boundary and (b) grain size distributions of sediment for initial riverbed materials.

4.2 Numerical simulations results for experimental channel

Figure 4 shows the sensitiveness of the riverbed slope on flow depth and riverbed evolution at equilibrium due to variations in channel geometry for a straight rectangular channel corresponding to calculation conditions as described in Table 1. The results suggest that steeper slope channel is highly influenced by the variable geometry comparing to mildly slope channel. The riverbed changes is more sensitive than that of the flow depth to channel geometry variations. Therefore, empirical relationship between water levels and corresponding discharge may perform well in mildly slope channels with normal flood when less sediment transport take place. However, the relationship may be affected considerably when active sediment transportation takes place during large flooding event or channels with steeper slope associated with channel variations such as rivers in mountainous region.

The numerical simulations results on the temporal variability in the water levels and discharge relationship for experimental channel with 1-D governing equations corresponding to the time series of flow discharge as of Figure 3(a) are shown in Figure 5. Figures 5(a) and (c) are results for channel narrowing considering the erodible and rigid riverbed, respectively. Figure 5(b) and (d) are simulated results as well with erodible and rigid riverbed, respectively where channel widening takes place. In all cases the initial riverbed is treated as flatbed. The results suggests that the relationship between water levels and discharge is varying according to riverbed evolution until reaching to quasi-equilibrium stage. The hysteresis effect is clearly visible in channel narrowing which exhibits the volume of water storage at the upstream of a location depend on the magnitude of riverbed evolution along with degree of the channel geometry variations. The water level and discharge relationship follows almost one to one unique relation in the case of numerical simulations with rigid bed condition. There also exists hysteresis effect but it is hidden and not visible evidently. The results also suggests that the relationship between water levels and discharge differ in relation to the variation in channel geometry, riverbed evolution and riverbed steepness. The riverbed evolutions are needed to take into consideration for formulating of rating curves where riverbed evolution takes place actively.

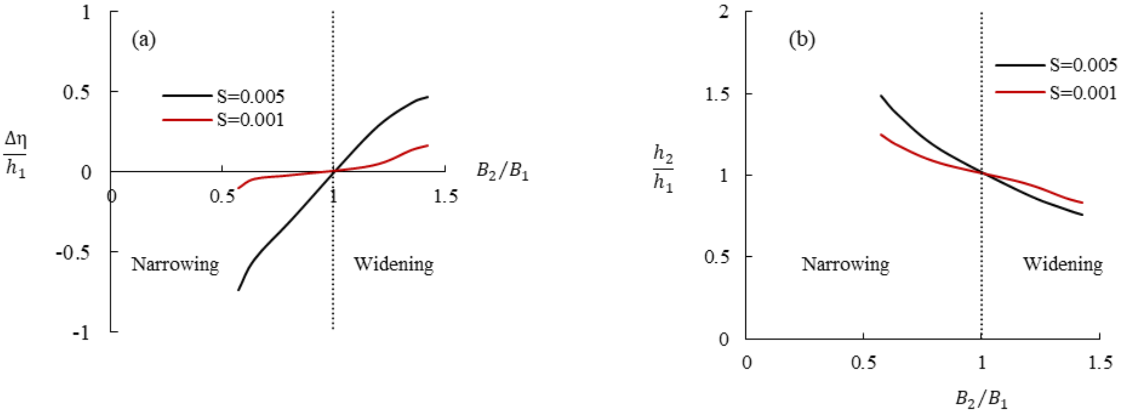

Figure 4. Flow depth and riverbed evolution corresponding to channel narrowing and widening based on 1-D numerical simulations. Input data: constant discharge Q=150 m^3/s, initial flatbed with bed slope =0.005 and 0.001, Manning's n=0.03, width ratios 0.6, 0.7 and 0.8 (for channel narrowing) and 1.2, 1.4 and 1.5 (for channel widening), exponent of non-dimensional bed shear stress, m=5/2.

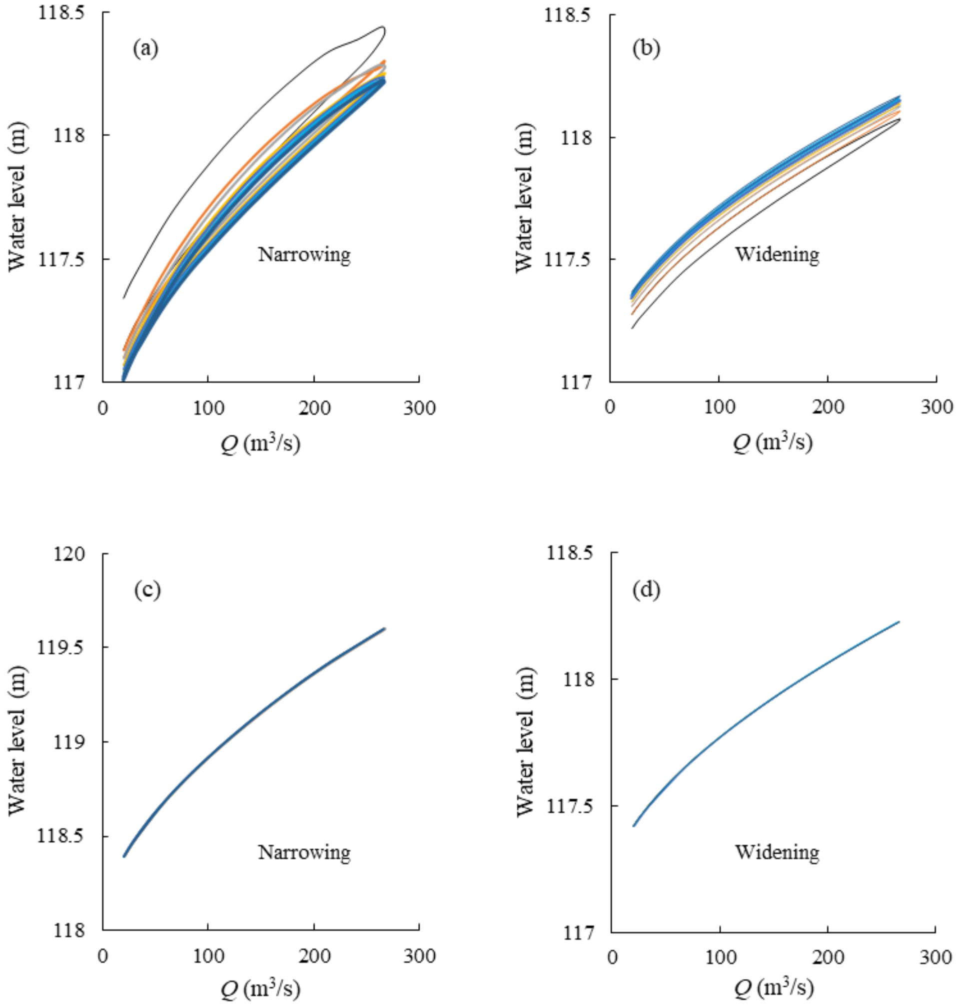

Figure 5. Stage-discharge relationship with experimental channel for rigid and erodible bed condition. Input data: time series of flow discharge as shown in Figure 4 (a), bed slope =0.005, Manning's n=0.03, width ratio 0.7 (channel narrowing) and 1.4 (channel widening), grain size distribution of riverbed material as shown in Figure 3(b), m=5/2.

4.3 Numerical simulations results for Seri River, Japan

Seri River originates in the Suzuka Mountain, Shiga prefecture, Japan and falls into the lake Biwa. The total length of the river is about 25 km with a basin area of 65 sq. km. Figure 6(a) is the study reach of Seri River showing the drainage network of the river basin. An upward concave curve characterizes the longitudinal riverbed elevation profile as illustrated in Figure 6(b). The riverbed slope is about 1/200 and there are no significant changes in the riverbed profile. The target area exhibits heterogeneous channel geometry in which alternate bars may formed. The river bed is composed of non-uniform sediment having grain-size distribution similar to Figure 3(b).

The variability in stage and discharge relationship curves for Seri, River Japan based on numerical simulations with 1-D governing equations under constant bedload supply at the upstream boundary are shown in Figure 7. Figure 7 (a) and (b) are located along section with channel widening. Figure 7 (c) and (d) are on the channel narrowing. The numerical simulation results exhibit upward and downward shifting of water level and discharge relationship corresponding to riverbed aggradation and degradation. The behavior of the curves are opposite in channel narrowing and channel widening. The computed results also clearly exhibit the hysteresis effect in both cases and is influenced by the degree of channel geometry variations.

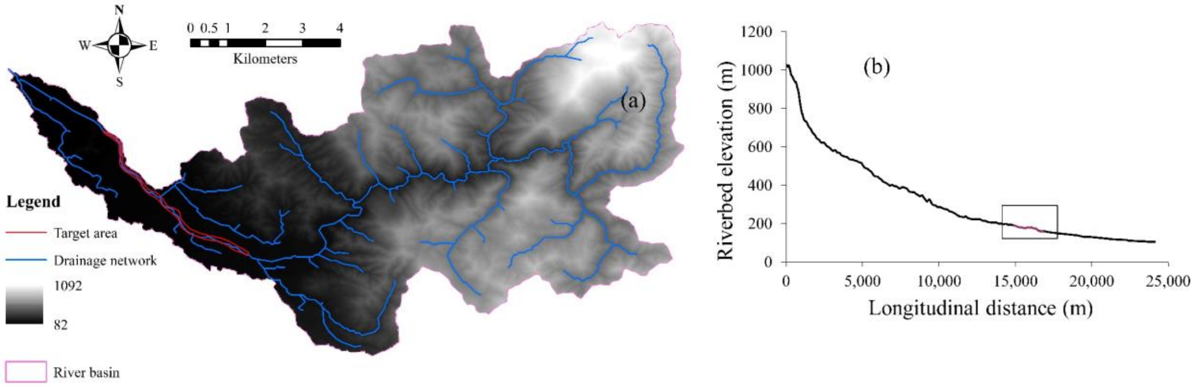

Figure 6. Characteristics of Seri River basin: (a) river basin with drainage network and (b) longitudinal riverbed profile.

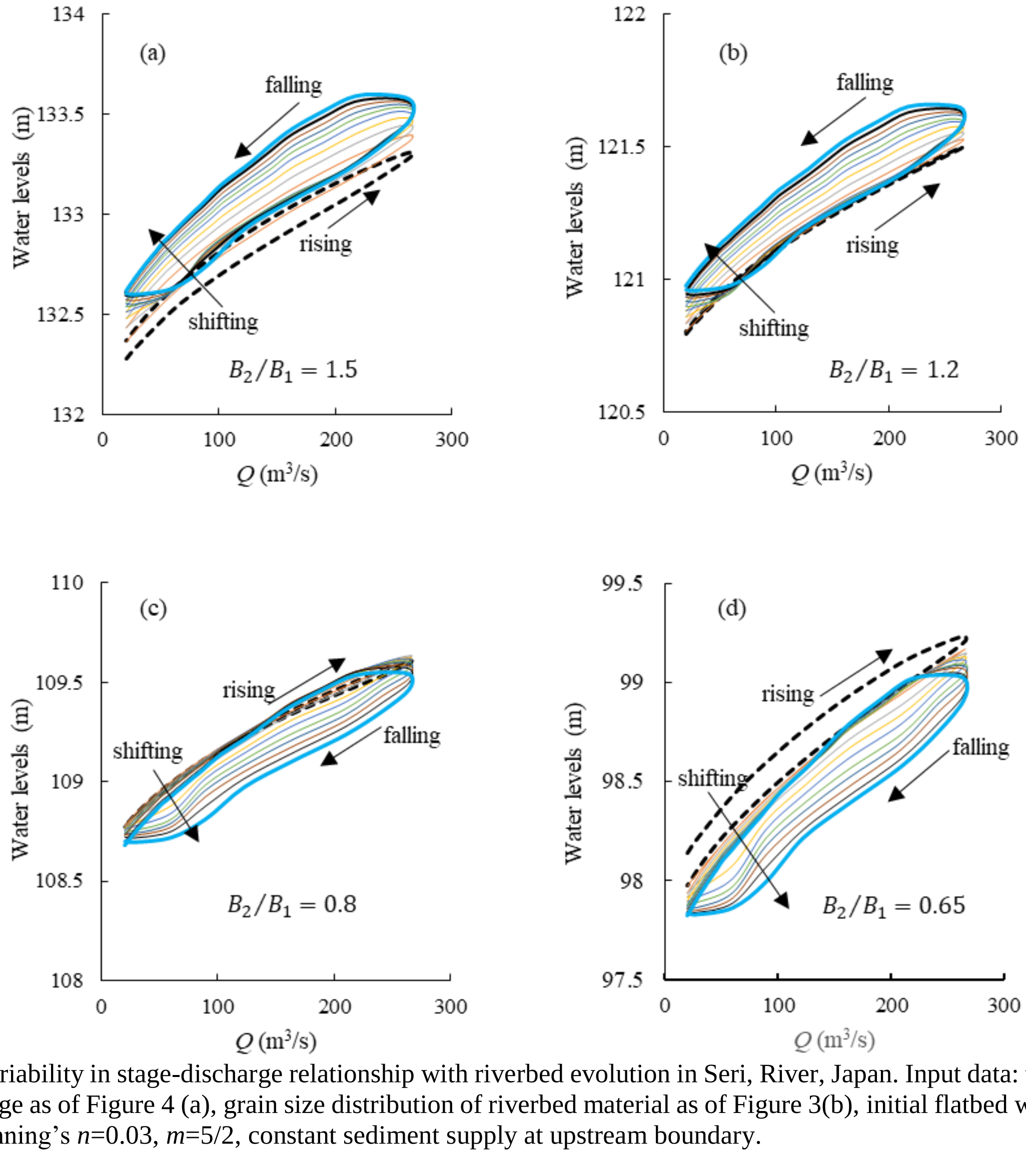

Figure 7. Variability in stage-discharge relationship with riverbed evolution in Seri, River, Japan. Input data: time series of flow discharge as of Figure 4 (a), grain size distribution of riverbed material as of Figure 3(b), initial flatbed with bed slope =0.005, Manning's n=0.03, m=5/2, constant sediment supply at upstream boundary.

5. CONCLUSIONS

The present study discusses variability in stage-discharge relationship under active riverbed evolution by means of numerical simulations with 1-D governing equations and based on regime equations. Bedload formula with non-dimensional bedload transport rate proportional to 2.5th power of the non-dimensional bed shear stress with bedload layer thickness is applied to evaluate riverbed evolutions. Based on the regime equations, the energy slope is steeper and milder for the channel widening and narrowing, respectively for m=5/2. This trend differs considerably for m=3/2. Stage-discharge relationship is sensitive to the variation of channel geometry as well as to the riverbed evolutions. The stage-discharge relationship curves react according to riverbed evolution and continue up to the attainment of quasi-equilibrium stage. Thereafter, riverbed together with the mean diameter and sediment size distributions of riverbed materials as well as stage-discharge relationship are not changing significantly.

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