APPLICATION OF HIGHER ORDER BATHYMETRY REPRESENTATION IN FIXED GRID SHALLOW WATER SOLVERS

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ABSTRACT

Fixed grid and flexible mesh solvers for the depth averaged Navier-Stokes equations (Shallow Water Equations) discretise a 2D domain into cells and faces. Many solution schemes consider the cells and faces to be flat-bottomed and either 'wet' or 'dry' depending on whether the water surface elevation exceeds the cell centre or face centre elevation. This first order representation causes errors in the results that depend on mesh size, and in the case of fixed grid solvers, mesh rotation angle, and in the case of flexible mesh solvers, mesh design. In this study we apply 'Sub-Grid-Sampling', a higher-order representation that considers 'partially wet cells' The mesh alignment and size sensitivities have been investigated by conducting numerical simulations of a U-shaped channel flume experiment, and a real-world meandering river with high quality of historical flood recordings. The benefits of 'Sub-Grid-Sampling' appear significant and far-reaching. At curved channels that are not mesh aligned, velocity fields no longer display "saw-tooth" patterns which cause artefact energy losses. Moreover, the sensitivities of results to mesh alignment and size are substantially reduced. This achievement is far-reaching in that fixed grid solvers with 'Sub-Grid-Sampling' can produce the same quality of results as a well-designed flexible mesh model or a cut-cell model.

Keywords: sub-grid-sampling, shallow water equation, flood modelling, grid alignment sensitivity

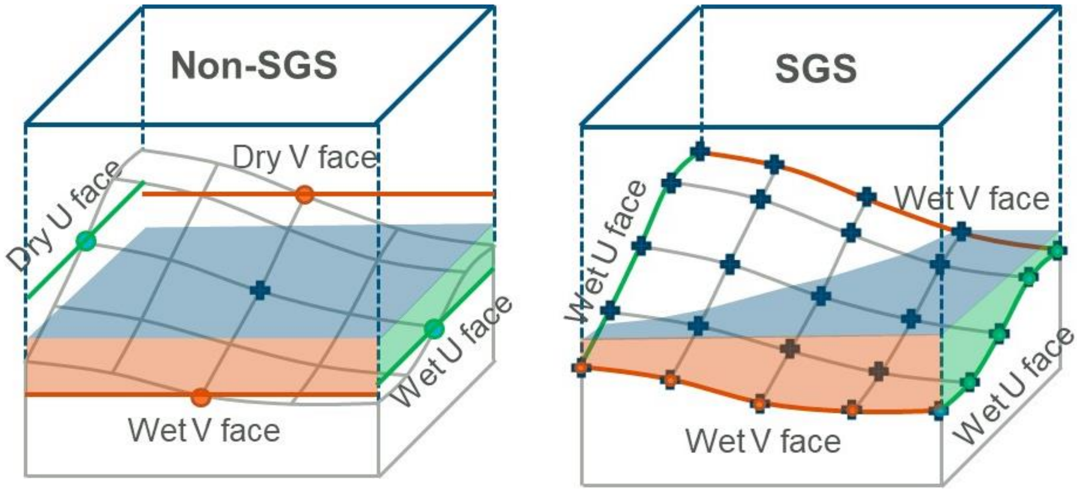
1. INTRODUCTION

When solving the 3D, or 2D depth averaged, Navier Stokes equations for fluid motion on a discretised domain, at some stage in the calculation process velocities and/or other prognostic variables must be interpolated to locations between their integration points. A range of interpolation schemes exist, and they are commonly classed according to their 'order', i.e. how the scheme's residual error varies with cell size. For a zero-order scheme the interpolation error is constant regardless of cell size. For a first order scheme the interpolation error is proportional to cell size: halving the cell size results in an interpolation error that is half as much. With a second order scheme the interpolation error is proportional to cell size squared: halving the cell size results in an interpolation error that is only a quarter as much. First order and second order interpolation schemes are most commonly used. First order schemes are usually unconditionally stable but produce a solution that is strongly dependent on mesh size, and also produce artefact diffusion – they tend to artificially smooth the solution. Second order schemes tend to be unstable, but once stabilised produce a solution that converges quickly as cell size is reduced and has less artefact diffusion – the solution retains more detail.

The majority of available software for performing 2D hydraulic modelling offer first and/or second order schemes with regard to interpolation of water depth and velocities, but most still treat the cells as 'flat bottomed' – i.e. a cell is either 'wet' or 'dry', with stored volume being cell area times depth. Similarly, the faces are treated as either wet or 'dry', with the available flux area being face length times depth. This is essentially a first order approach, and the errors introduced into the solution by this approach are very significant for two reasons. Firstly, the solution retains a significant dependency on mesh size despite using second order schemes for other variables, and secondly, misalignment between the mesh and wet/dry boundaries causes artefacts in the velocity field and consequently the solution displays significant dependency on mesh-alignment. The mesh-alignment issue can be overcome by using a flexible mesh fitted to the flow path, but the problem emerges again once the primary flow path breaches its banks and the wet-dry boundary moves. The better solution, applicable to both fixed grid and flexible mesh solvers, is to adopt a second order treatment for cell storage and face flux – in other words to consider 'partially wet' cells and faces.

2. SUB-GRID-SAMPLING

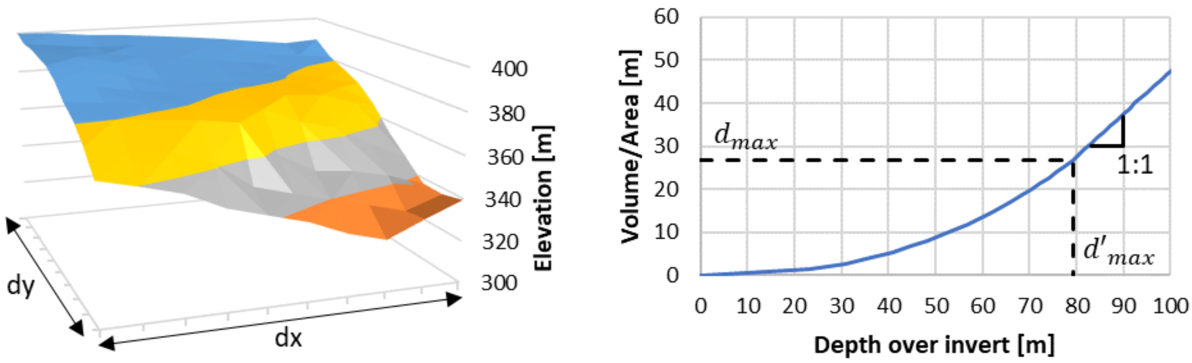
Partially wet cells and faces may be considered by sampling the digital elevation model (DEM) at a number of points within a cell and computing the water surface elevation as a function of cell stored volume. The sampling process is shown conceptually in Figure 1, where it can be seen that without sub-grid-sampling (SGS) the cell is only hydraulically connected on two faces, but with SGS it is hydraulically connected on all four faces.



(a) (b)

Figure 1. Regular 2D Mesh Terrain Sampling. (a) Traditional approach of a single elevation per cell centre and cell face. (b) Sub-Grid Sampling (SGS) at a higher resolution over the cell and across each cell face.

The relationship between water depth over cell invert (the lowest point within the cell) may be stored for each cell as a discretised look-up table or else a parameterised smooth function. An example cell DEM and its associated parameterised smooth function relating depth over invert, d', to cell average depth, d, are shown in Figure 2. The function need only be defined for water elevations between the lowest and highest elevations within the cell, once the entire cell is wet a 1:1 storage slope may be assumed. A similar approach may be adopted for faces, where the function relates depth over face invert to average face depth.



(a) (b)

Figure 2. SGS implementation. (a) Example cell sub-grid elevation, (b) Cell storage function.

The sub-grid-sampling process and implementation of the associated parameterised smooth functions for cells and faces has been integrated into the TUFLOW HPC 2D hydraulic solver, Collecutt and Syme (2017), which was used to produce the numerical results presented. For the analyses presented the 2020-01-AA software build (BMT, 2020) was used with no changes to the default parameters.

3. RESULTS

3.1 Smooth U-Bend

Engineered deep sided channels are common in urban environments and provide an excellent example of interaction between velocity field and the misalignment between mesh and wet/dry boundary. The U-bend flume of De Vriend and Koch (1978) is illustrated in Figure 3.

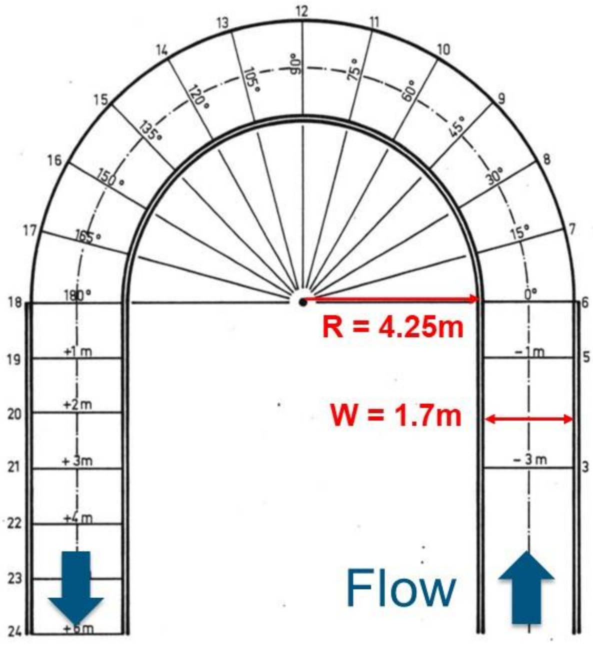


Figure 3. U-Bend flume

The channel is 1.7 m wide and has an inside radius of 4.25 m. Water levels along the inside, centreline and outside were measured every 1 m in longitudinal length or at 15° intervals around the bend. The flow applied was 0.189 m^3/s with a downstream water depth of 0.18 m. The experiment flume was reported to be ‘smooth’, and the roughness height is documented as 0.001 to 0.0005 in a published 3D numerical model verification report (Wang et al., 2009). Based on these values, a Manning’s n value of 0.0125 can be derived and was used in this study. Figure 4 compares model and flume results for the steady state flow condition without sub-grid-sampling. Artefacts in the water surface elevation are visible in Figure 4 (a), and the velocity field near the wet/dry boundary in Figure 4 (b) shows the disturbances caused by a first order treatment. Consequently, the overall drop in water elevation around the bend, Figure 4 (c), is significantly overestimated by the numerical solution.

(a) Water surface elevation

(b) Zoomed view along wet/dry boundary

(c) Simulated vs recorded results

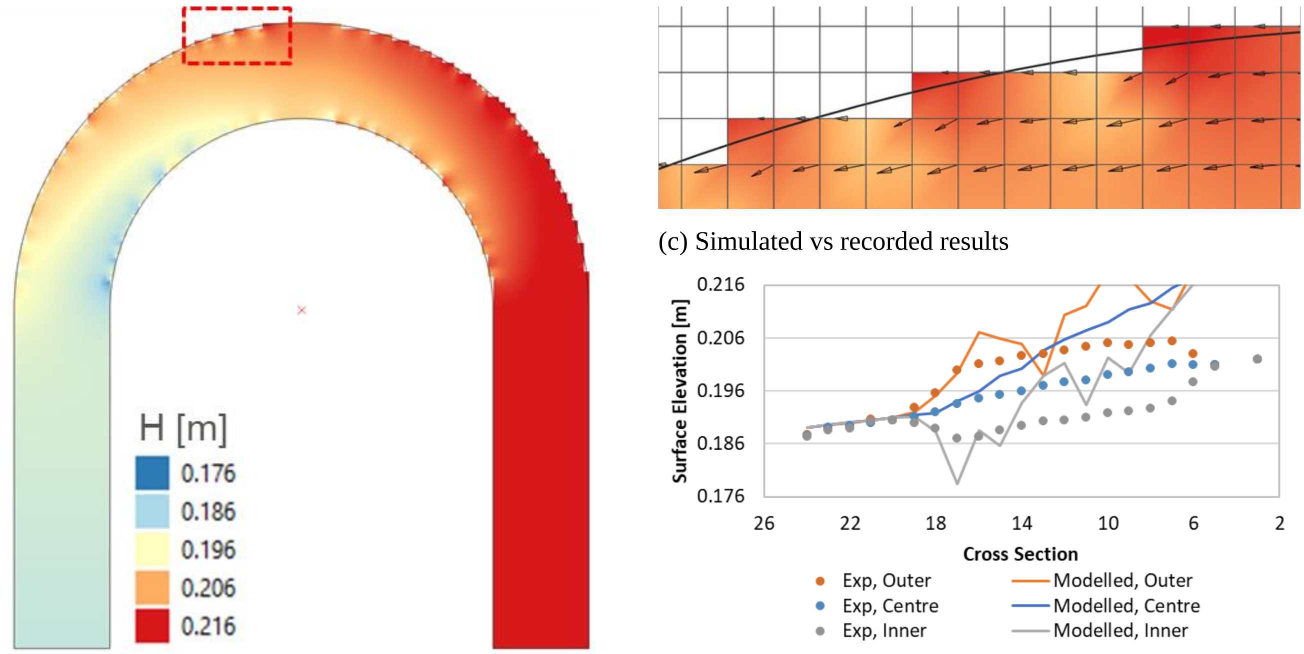


Figure 4. U-Bend result, without sub-grid-sampling

The same model repeated with the sub-grid-sampling is shown in Figure 5. Note the red boxed cells in Figure 5 (b) that are now only ‘partially wet’. The artefacts in the water surface elevation and velocity vectors near the dry wall are no longer present, and the overall drop in water elevation around the bend now closely matches the test result.

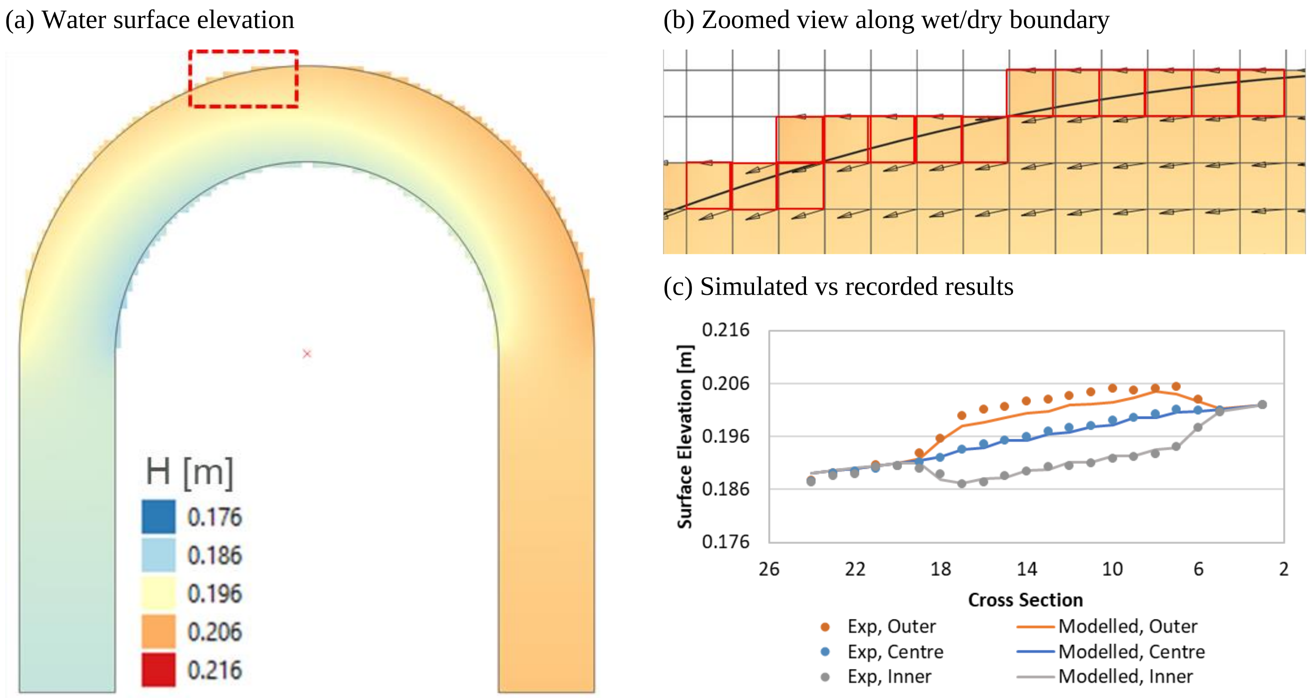


Figure 5. U-Bend result, with sub-grid-sampling

3.2 Brisbane River

The Brisbane River provides an excellent example of a large natural waterway with tight meanders. It also has a high quality bathymetry data set and recorded flows and levels during the 2011 flood (BMT 2015). During the flood peak, the in-bank flow through central Brisbane city was 9000 m^3/s. The model bathymetry and the locations of the gauge peaks and water marks taken from the 2011 flood are presented in Figure 6 for an 11 km stretch of the river. To reproduce the water levels during the peak of the 2011 flood, a constant inflow rate, a fixed downstream water level of 2.7m, and a Manning's *n* of 0.026 were applied in the simulations. The mesh size was changed from 50m to 10m to investigate the mesh size convergency of with/without the SGS method.

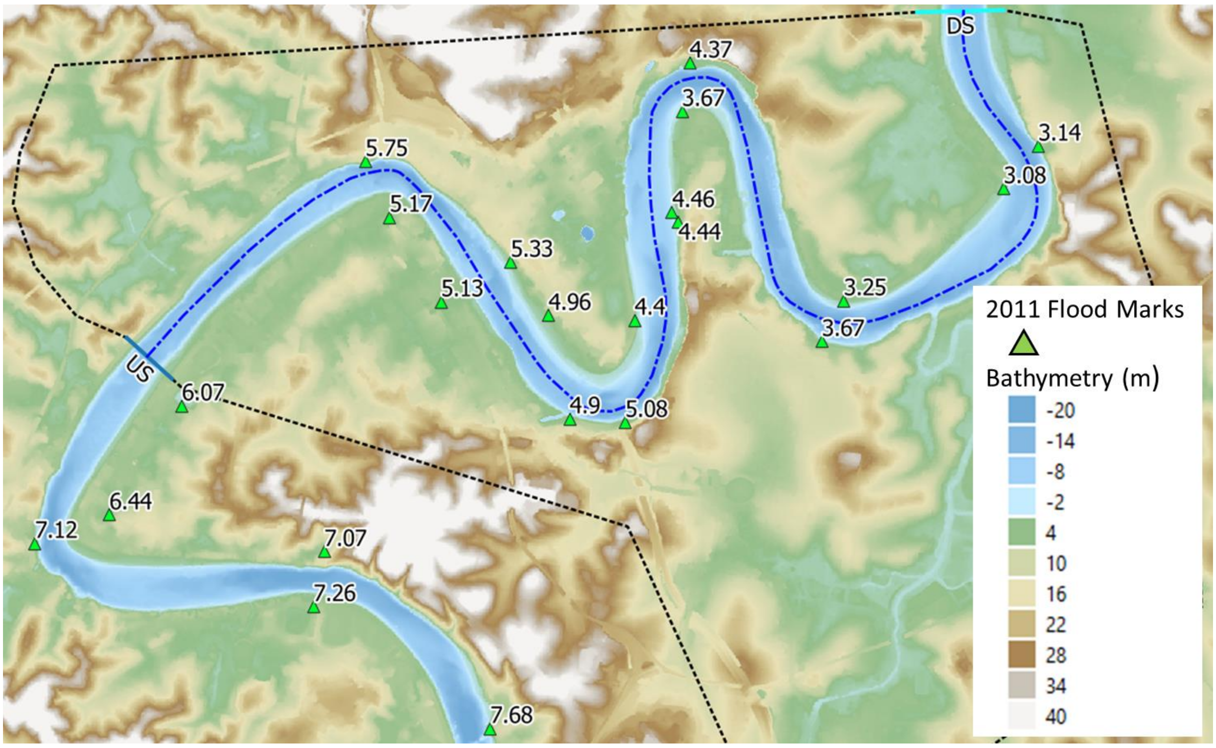


Figure 6. Brisbane River 5m grid bathymetry and the water marks during the 2011 flood

3.2.1 *Head loss along the Brisbane River*

Figure 7 shows the simulated water surface level using 30m grid SGS model, and Figure 8 presents the modelled water level along the streamline (blue dash dot line in Figure 7). The modelled water levels at the flood mark locations are also plotted (green squares) and compared with the recorded flood mark levels (black triangles). The modelled water level agreed well with the historical flood mark levels. A super elevation of 0.8m was observed at the Story Bridge bend, and this was also reproduced well by the model.

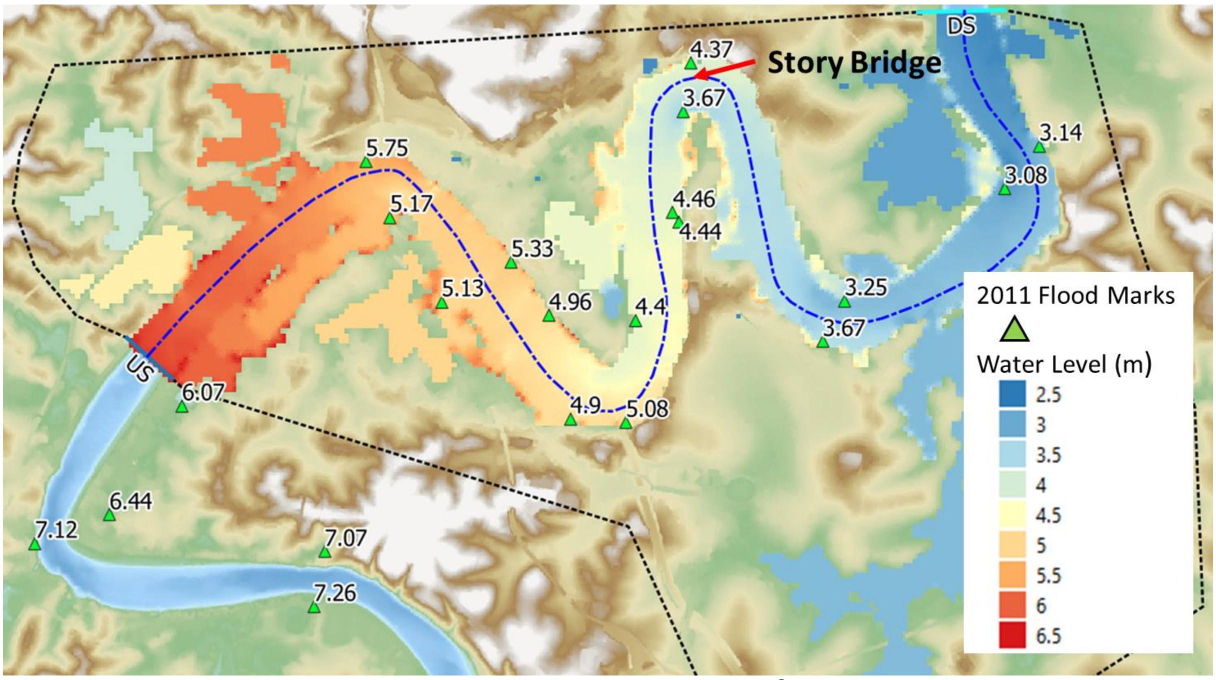


Figure 7. Simulated water surface level under 9000 m^3/s flow vs 2011 flood

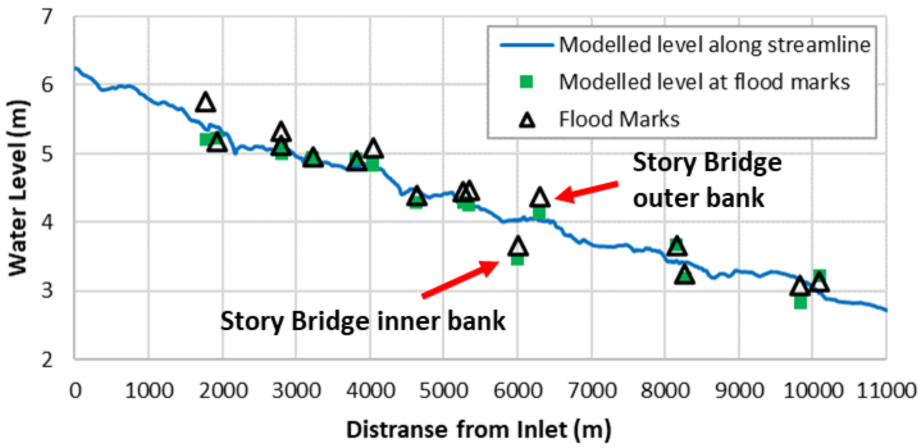


Figure 8. Simulated water surface along the streamline of Brisbane River compared with the flood mark

3.2.2 *Mesh size sensitivity*

Several factors contribute to mesh size sensitivity within the model. In particular, the eddy viscosity model if not correctly posed can become a source of mesh size dependency, Collecutt et. al. (2020). The model was solved for mesh sizes ranging from 10 m up to 50 m and the predicted head loss along the reach recorded. The results, without and with sub-grid-sampling, are shown in Figure 9. The mesh size sensitivity in the results is greatly reduced with the use of sub-grid-sampling. The difference in total head loss was only 0.25 m (less than 10%) between the 10m and 50m SGS model, whereas the non-SGS model showed a difference of 0.94m. The reduction in mesh size sensitivity has two significant repercussions - firstly, model errors introduced through changes in cell size without re-calibration are much reduced, and secondly the calibration process may proceed at a coarser resolution (with associated faster solve times), with final calibration at production mesh size becoming a formality rather than a protracted process.

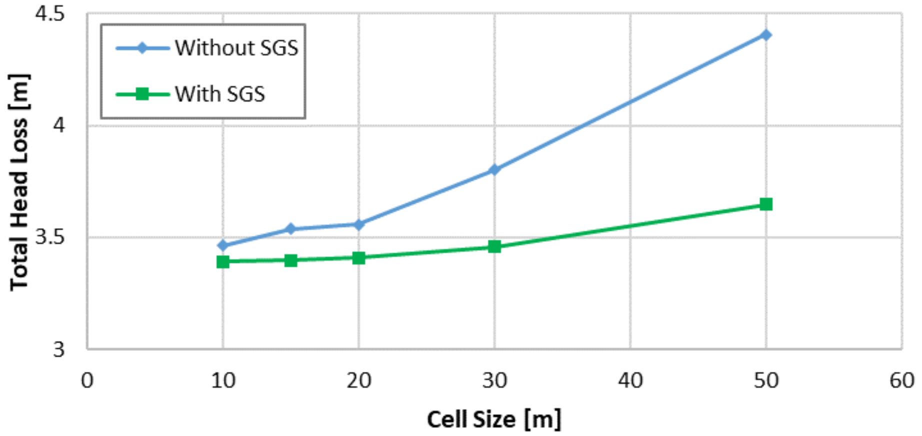


Figure 9. Brisbane River head loss with and without sub-grid-sampling

3.3 Hydrologic Response

Kitts et. al. (2020) discuss the application of sub-grid-sampling regarding the hydrologic response for whole catchment models with direct rainfall applied. With every cell face in the model correctly capturing the lowest elevation along its length (to the sub-grid-sampling resolution), the approach can represent sub-cell scale flow paths. Consequently, artefact water retention along tributaries (a known problem for fixed-grid solvers) is eliminated, and the catchment response is significantly less dependent on mesh size.

For example, Figure 10 shows the flow hydrographs for a Quadtree direct rainfall whole of catchment model of the Tamar River catchment in Tasmania using two resolutions. The Hi-Res Quadtree mesh uses 160, 80, 40 and 20 m cell sizes while the Lo Res mesh halves these cell sizes, i.e. 80, 40, 20 and 10 m. The grey and yellow hydrographs are for without SGS and their marked difference in peak flow, shape and timing demonstrate significantly different results between the Hi and Lo resolutions, causing a cell size convergence test failure and the need for further refinement of the cell sizes (and much longer solve times) to achieve convergence. The coarser resolution simulation (yellow hydrograph) shows greater attenuation and lower volume passing the gauge site compared to the Hi-Res finer resolution simulation due to the coarser cell sizes congesting flow paths and retaining water within the catchment. In contrast, the blue and orange hydrographs are with SGS applied and show much less flow attenuation and water retention. The two SGS simulations also produce very similar results between the two resolutions, thereby demonstrating excellent cell size convergence and the ability to use the faster running Lo-Res model for day-to-day modelling.

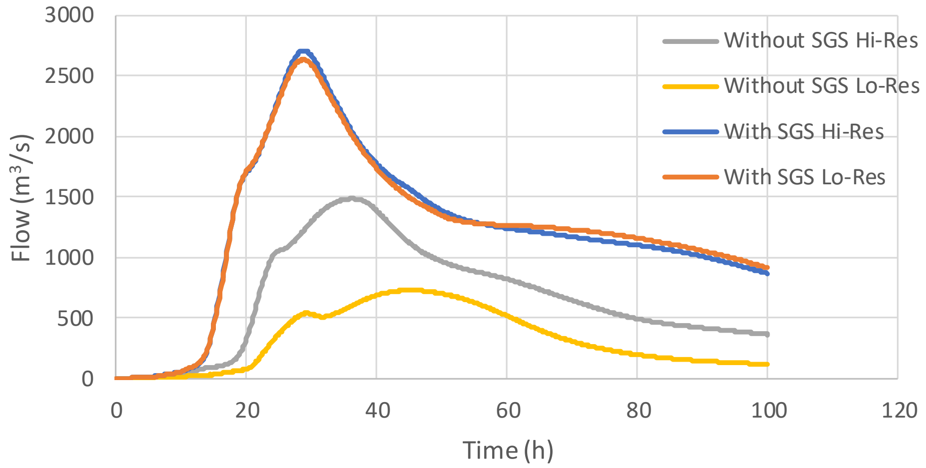


Figure 10. Direct rainfall whole of catchment cell size convergence with and without sub-grid-sampling

4. CONCLUSIONS

The benefits of using a second-order approach for cell volume storage and face flux area are many. Firstly, velocity fields near wet/dry boundaries are greatly improved. Secondly, artefact energy losses associated with incorrect velocity fields near wet/dry boundaries are practically eliminated. Thirdly, overall mesh size and mesh alignment sensitivities within the model are significantly reduced. And fourthly, for whole of catchment direct rainfall models, artefact water retention is reduced, and catchment response greatly improved.

The implications of these achievements cannot be understated – they collectively represent a significant step forward in reducing model error and improving the speed and efficiency with which accurate and calibrated hydraulic models can be constructed.

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