IMPROVEMENT OF THE MIXING LENGTH MODEL AND NUMERICAL CALCULATION OF THE FLOW OVER A BACKWARD-FACING STEP

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ABSTRACT

Among turbulence models for closure of Reynolds averaged equation based on Boussinesq eddy viscosity hypothesis, the standard k-epsilon model has been widely applied in engineering. However, the model mainly considers the effects of turbulence transport and dissipation on Reynolds stress in small-scale structure, which leads to low accuracy in the calculation of flows such as rotational flow, adverse pressure gradient and back flow. Prandtl mixing length model is the simplest algebraic equation model, but it contains the velocity gradient, which in some sense corresponds to the large-scale structure of turbulence. In this paper, based on the comparative study of the two models, an improved mixed turbulence model is firstly proposed by means of dimensional analysis, in which the calculation formula of eddy viscosity coefficient includes not only turbulent kinetic energy k and its dissipation rate ε, but also the velocity gradient of the mean field. Then the model is applied to the numerical simulation of the flow over a backward-facing step. And the calculated longitudinal velocity profile, turbulent kinetic energy k and dissipation rate ε distribution of separation zone are analyzed. Meanwhile, by comparing with some measured data and the calculation results under the standard k-ε model, it is fortunately found that in the range of Reynolds number (Re) from 5×10^3-1×10^5, the improved mixed turbulence model with velocity gradient can predict this type of turbulent flow more accurately.

Keywords:Turbulence scale; mixed turbulence model; Reynolds stress; flow over a backward-facing flow; numerical calculation

1. INTRODUCTION

Since O. Reynolds used the statistical method to average N-S equation and further created the turbulence theory (Reynolds, 1895), some problems about the closure of Reynolds stress in the RANS equation has not yet been solved perfectly. In the past century, based on the Boussinesq eddy viscosity hypothesis (Boussinesq, 1877), scholars have conducted a lot of researches on turbulence models. Among them, the Prandtl mixing length model and the standard k-epsilon model have been successfully applied and verified (Prandtl, 1925; Launder et al, 1972). In detail, on the one hand, the mixing length model, by analogy with the motion of gas molecules, proposes the concept of mixing length of particles or vortices in turbulence, which relates the turbulent viscosity coefficient to the velocity gradient of the mean field. In other words, the turbulent eddy viscosity is considered to be mainly related to the random collision mixing of turbulent vortex and changes in the velocity strainrate of the mean field. However, due to the failure of mixing length model theory in considering the transport characteristics of pulsating flow field parameters (such as turbulent kinetic energy k), this model is only suitable for some simple flows, such as pipes, open channel uniform flow, boundary layer flow, etc. On the other hand, the standard k-ε model is based on Kolmogorov theory of local homogeneous and isotropic turbulence (Karman, 1930), and proposes a calculation method for the turbulent eddy viscosity coefficient, the turbulent kinetic energy k and its dissipation rate ε, which fully considers the effects of the turbulence transport and its dissipation on the Reynolds stress in small-scale structures. And it has been confirmed by a great deal of engineering practices that the k-ε model can be applied to calculating some complex turbulence flows, such as the non-buoyant plane jets, flat wall boundary layer flow, channel flow, spinless or weak swirling backflow, etc, but its prediction accuracy of the flow near the boundary layer separation point, large curvature flow, non-circular cross-section pipe flow, open channel flow and so on is poor.

In this paper, by a comparative study of the two models, an improved mixed turbulence model is firstly proposed using a dimensional analysis method, in which the calculation formula of the turbulent eddy viscosity coefficient includes not only the turbulent kinetic energy k and its dissipation rate ε, but also the velocity gradient of the mean field. Then this model is used to numerically simulate the two-dimensional turbulent flow over a backward-facing step, and the separation zone length, streamwise velocity profile as well as turbulent characteristics are discussed in detail.

2. COMPARISON AND IMPROVEMENT OF TURBULENCE MODELS

2.1 Comparison of Prandtl mixing length model and standard k-ε model

For the two-dimensional parallel shear turbulent flow, according to the Boussinesq eddy viscosity hypothesis, the turbulent shear stress for the Prandtl mixing length model can be described as

$$\tau_t = -\rho\overline{u_x' u_y'} = \rho l^2 \left|\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right| \cdot \frac{\mathrm{d}\bar{u}}{\mathrm{d}y} \tag{1}$$

Combining the Boussinesq eddy viscosity assumption, the turbulent eddy viscosity coefficient for the mixing length model can be expressed as

$$\nu_t = l^2 \left|\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|, \quad l = ky \tag{2}$$

where l is the mixing length. And the formula of it is obtained by Karman (von Karman, 1930) through experiments, in which y is the normal distance from the wall, k is a constant, and $k \approx 0.41$.

In the standard k-ε model, the turbulent eddy viscosity coefficient is the function of the turbulent kinetic energy k and its dissipation rate ε, which is written as

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \tag{3}$$

where C_μ is empirical constant.

Besides, turbulent kinetic energy k and its dissipation rate ε also satisfy the following differential equations in transport form, which can be expressed as

$$\frac{\partial k}{\partial t} + \bar{u}_l \frac{\partial k}{\partial x_l} = \frac{\partial}{\partial x_l}\left(C_k \frac{k^2}{\varepsilon}\frac{\partial k}{\partial x_l}\right) - \varepsilon + P \tag{4}$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_l \frac{\partial \varepsilon}{\partial x_l} = \frac{\partial}{\partial x_l}\left(C_\varepsilon \frac{k^2}{\varepsilon}\frac{\partial \varepsilon}{\partial x_l}\right) + C_{\varepsilon 1}\frac{\varepsilon}{k}P - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{5}$$

where

$$\varepsilon = 2\nu\overline{\frac{\partial u_i'}{\partial x_l}\frac{\partial u_i'}{\partial x_l}}, \quad P = -\overline{u_i' u_l'}\frac{\partial \bar{u}_i}{\partial x_l} \tag{6}$$

The empirical constants values in Eqs. (3), (4) and (5) are shown in Table 1.

Table 1. Empirical parameter values in k-ε model

C_μ	$C_{1\varepsilon}$	$C_{2\varepsilon}$	σ_k	σ_ε
0.09	1.44	1.92	1.0	1.3

Comparing with the mixing length model, the standard k-ε model not only includes the transport of the pulse velocity scale ($k^{1/2}$), an important turbulent characteristic parameter, but also indirectly considers the transport of the turbulent length scale through the transfer and dissipation of turbulent kinetic energy based on the energy cascade principle of the turbulent vortex. And this is believed to be an important reason why the k-ε model can successfully predict many shear-type turbulences. Further, combined with the disadvantage of the Prandtl mixing length model, that is, the uncertainty of the mixing length l in complex flows, we are to express the mixing length l as the function of the turbulent kinetic energy k and its dissipation rate ε. Thus, an improved mixed turbulence model can be proposed in which the turbulent eddy viscosity coefficient is not only the function of the turbulent kinetic energy k and its dissipation rate ε but also closely related to the change of the velocity gradient of the mean flow field. Particularly, the velocity gradient of the mean field can be regarded as an effect associated to the large-scale vortex of turbulence, which is not fully considered in the standard k-ε model.

2.2 Improved mixed turbulence model

Assuming that the turbulent eddy viscosity coefficient is the function of turbulent kinetic energy k, turbulent kinetic energy dissipation rate ε, and the velocity gradient of the mean flow field, we can achieve

$$\nu_t = f(k, \varepsilon, \frac{d\overline{u}}{dy}) \tag{7}$$

Performing a dimensional analysis on Eq.(7), we can obtain the following dimensionless number as

$$\prod = [\nu_t][k]^{\alpha}[\varepsilon]^{\beta}[\frac{d\overline{u}}{dy}]^{\gamma} \tag{8}$$

According to the dimensional harmony principle, the power exponents are calculated as α=3, β=2, γ=1.

Substituting the power exponents into the above formula and considering the two-dimensional characteristics of the flow, the improved expression of the turbulent eddy viscosity coefficient is

$$\nu_t = C\frac{k^3}{\varepsilon^2}\left|\frac{\partial\overline{u}}{\partial y} + \frac{\partial\overline{v}}{\partial x}\right| \tag{9}$$

where C is the proportional coefficient. Eq.(9) together with the aforementioned turbulent kinetic energy k equation Eq.(4) and the dissipation rate ε equation Eq.(5), the improved mixed turbulence model is formed. The method of calculating the C value is to compare the expression of the improved turbulent eddy viscosity coefficient with that of k-ε model, that is,

$$\nu_t = C\frac{k^3}{\varepsilon^2}\left|\frac{\partial\overline{u}}{\partial y} + \frac{\partial\overline{v}}{\partial x}\right| = C_{\mu}\frac{k^2}{\varepsilon} \tag{10}$$

and further

$$C = C_{\mu} \Big/ \left(\frac{k}{\varepsilon}\left|\frac{\partial\overline{u}}{\partial y} + \frac{\partial\overline{v}}{\partial x}\right|\right) \tag{11}$$

The calculated results of the k-ε equation model for the fully developed two-dimensional incompressible fluid turbulent flow between parallel plates are taken as the initial data for calculation of C value. The calculated values of k, ε and the velocity strain rate on each node of the same profile are substituted into Eq. (11) to calculate the C values of each node. Then, taking the average on this profile, and we can get the empirical value of the proportional coefficient C, and C is about 0.01.

3. CALCULATION AND ANALYSIS OF THE TWO-DIMENSIONAL TURBULENT FLOW OVER A BACKWARD-FACING STEP

3.1 Calculation zone and boundary conditions

The schematic diagram of the calculation area is illustrated as Figure 1 with a step height of 22mm. The heights before and after the step are 66mm and 88mm respectively. The length of the calculation zone is 220mm before the step with 1200mm after the step.

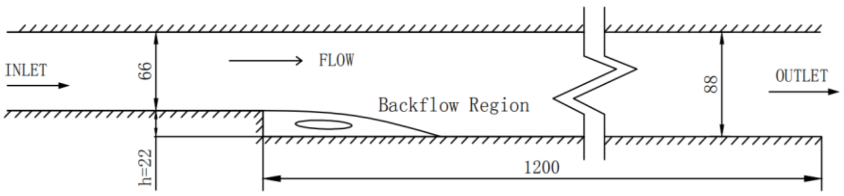


Figure 1. Schematic diagram of calculation zone (unit: mm)

The boundary condition of the inlet is a given incoming flow velocity, which is taken as a fully developed two-dimensional turbulent flow velocity distribution between parallel flat plates. The outlet boundary is a free flow, 1.2m away from the step; the upper and lower boundaries are non-slip solid walls; the turbulence intensity is 5%, and the turbulent viscosity ratio is 10. In the calculation area, the grid types are all quadrangular, the grid number is totally 118854, and the grid independence check is performed.

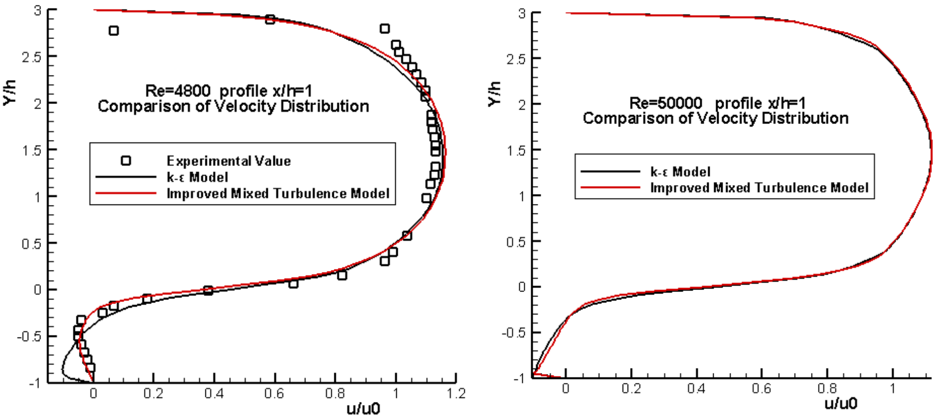
3.2 Analysis of calculation results

3.2.1 Longitudinal velocity distribution

Figure 2 shows the simulation results of the longitudinal velocity distribution in the separation zone over a backward-facing step at a step height h downstream of the step (i.e. profile x/h=1) under the conditions of Re=4800 and Re=50000. Here the Reynolds number is defined as

$$\mathrm{Re} = \frac{uh}{\nu} \tag{12}$$

where u is the average velocity of the inlet section, h is the step height, νis the molecular kinematic viscosity coefficient.



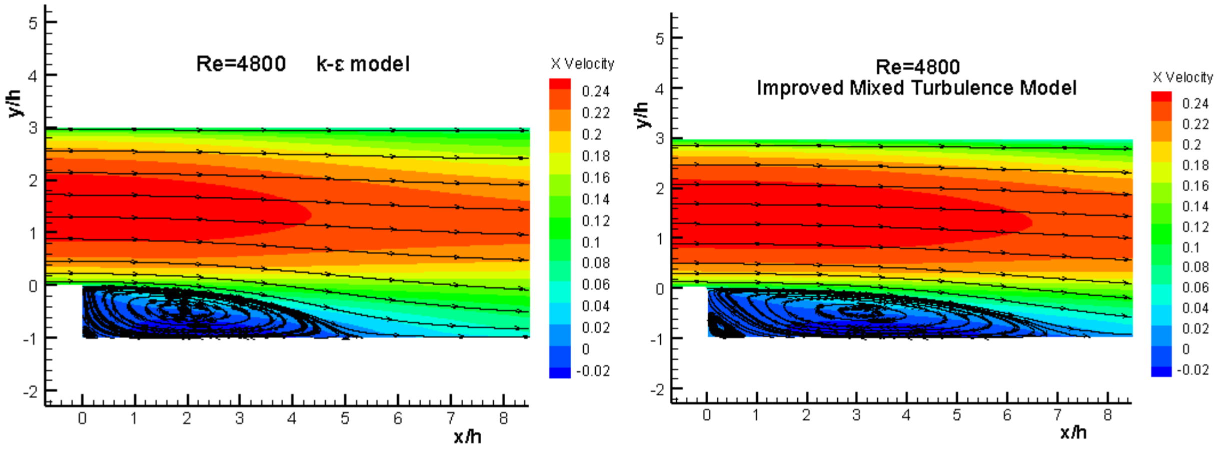
(a) Re=4800 (b) Re=50000

Figure 2.Longitudinal velocity profile at x/h=1 section

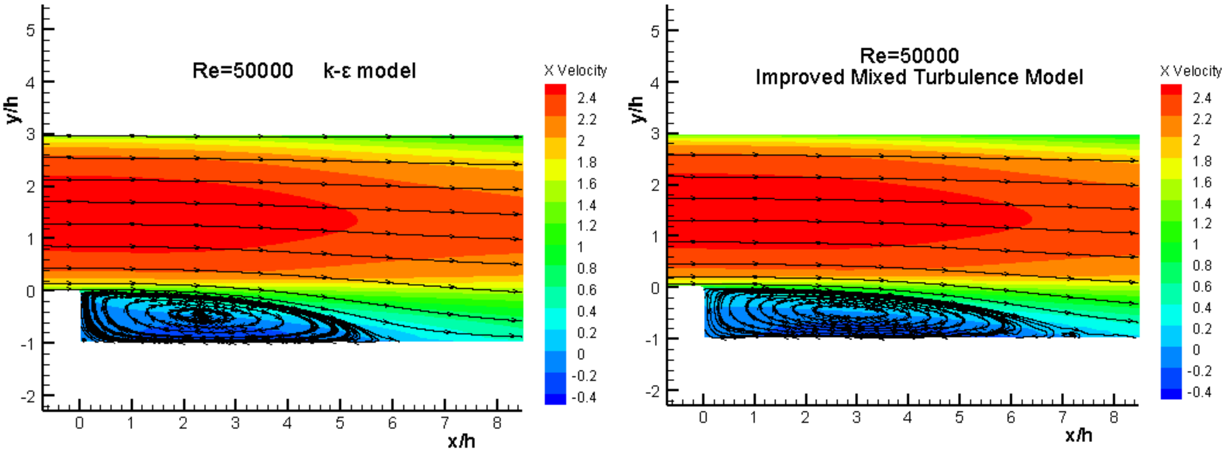
As is shown from the comparison in Figure 2(a), when Re=4800, the calculation result of the improved mixed turbulence model is closer to the experimental data (Armaly et al, 1983; Xiao et al, 2013), that is to say, the improved mixed turbulence model including the velocity gradient can obtain the velocity profile closer to the experimental data than the standard k-ε model. However, when Re=50000, the velocity distribution obtained by the improved mixed turbulence model and the standard k-ε model is basically the same, as Figure 2(b) shows.

3.2.2 Characteristics of flow field and separation zone

The calculation results of the streamline and vortex separation zonenear the step are shown in Figure 3.



(a) Re=4800



(b) Re=50000

Figure 3. Comparison of streamline and vortex separation zones near the step.

Relevant studies (Xiao et al,2013) show that when Re =4800, the experimental value of the ratio of the length of the separation zone to the height of step is about x/h =6. Comparing the simulation results of the two models, the length of separation zone calculated by the k-ε model is smaller than the experimental value, which is about 5.0, but the result obtained by the improved mixed turbulence model is larger than the experimental value, which is about 6.9. It can be seen that both are within the acceptable range.

When Re =50000, it can be seen from the simulation results that the length of separation zone obtained by the two models is about one step height difference, and the separation zone length obtained by the improved mixed turbulence model is larger. Moreover, relevant studies (Armaly et al, 1983) show that there is additional separated-flow region downstream of the separation zone when the Reynolds number is large. The improved mixed turbulence model successfully simulates this separated-flow region as figure 4 shows, which is undoubtedly an important advance.

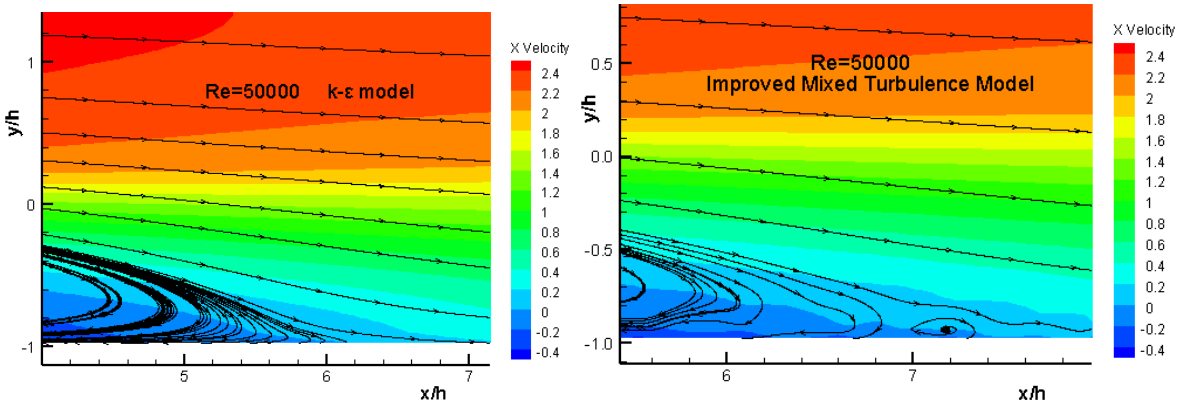
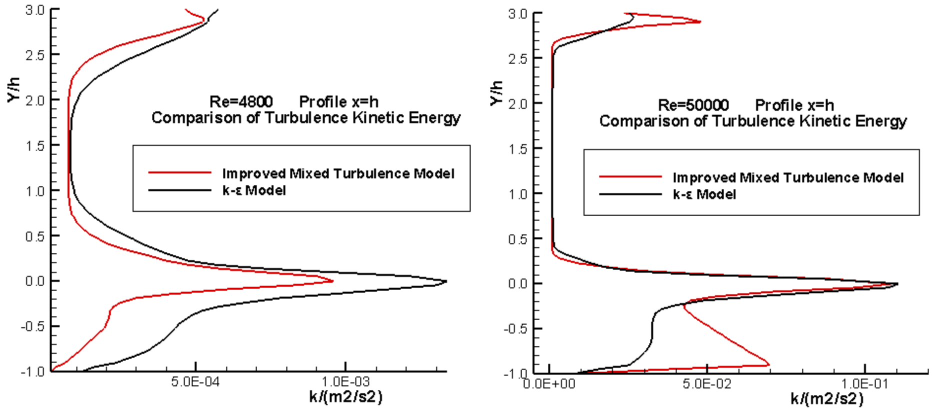


Figure 4.Local magnification of additional separated-flow regions.

3.2.3 Characteristics of turbulent kinetic energy and dissipation rate

Turbulent kinetic energy can represent the strength of flow turbulence. The greater the turbulent kinetic energy is, the more violent the flow turbulence is, and the greater the fluid collision and friction are. Figure 5 shows the distribution of turbulent kinetic energy at a step height h downstream of the step (i.e. x/h=1) when Re =4800 and 50000.

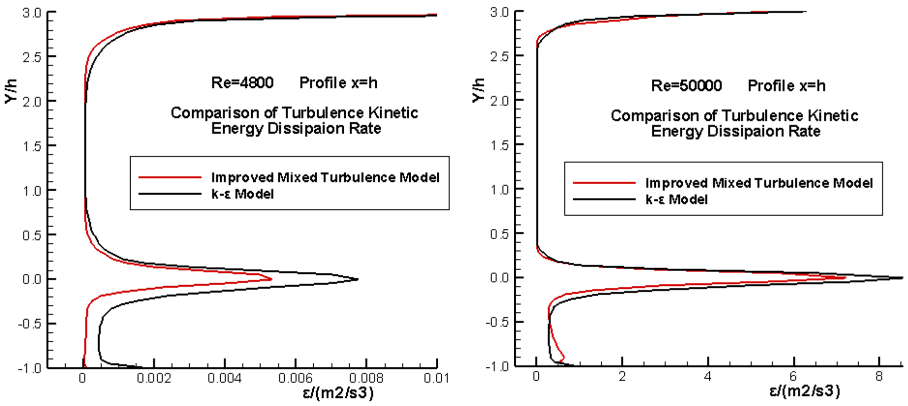


(a) Re=4800 (b) Re=50000

Figure 5. Distribution of turbulent kinetic energy at the profile x/h=1.

In Figure 5a, when Re =4800, the calculated turbulent kinetic energy of the mixed turbulence model is smaller than that of the k-ε model, while the two trends are consistent. However, when Re=50000 (Fig. 5b), at the range of y/h=－0.2-0.6, the turbulent kinetic energy obtained from the calculation of the two models is comparable. In the separation zone below the step, the turbulent kinetic energy increases significantly due to the separation vortex. And near the wall, the strength of turbulence is obviously reduced by the viscous layer, which makes the turbulent kinetic energy decay sharply. As is seen in figure 5b, the turbulent kinetic energy distribution calculated by the mixed turbulence model has obvious peak near the wall (y/h≈－0.9 and y/h≈ 2.9) and then decreases sharply. This is a reasonable result consistent with the theoretical analysis of flow physics, indicating that the improved model is superior to the k-ε model in the simulation of the turbulent characteristics (the turbulent kinetic energy k) near the boundary.

Turbulent kinetic energy dissipation rate ε represents the rate at which the mechanical energy of isotropic small-scale eddies is converted into heat. The larger the dissipation rate is, the greater the energy losses. Figure 6 shows the distribution of turbulent kinetic energy dissipation rate at a step height h downstream of the step (i.e. x/h=1) when Re=4800 and 50000.



(a) Re=4800 (b) Re=50000

Figure 6. Distribution of turbulent kinetic energy dissipation rate at the profile x/h=1.

As is shown in Figure 6, when Re=4800 and 50000, the trend of the turbulent kinetic energy dissipation rate ε distribution obtained from the simulation of the two models is consistent. Particularly, when Re=4800, the calculation result of the mixed turbulence model is smaller than that of the k-ε model, but the results of the two is almost the same when Re=50000, which is also consistent with the distribution of the turbulent kinetic energy k in Figure 5. Therefore, the turbulent kinetic energy dissipation rate ε calculated by the improved mixed turbulence model is credible.

4. CONCLUSION

In this paper, through a comparative study of the standard k-ε model and the Prandtl mixing length model, for the uncertainty of the mixing length *l* in complex flows, we consider expressing *l* as a function of the turbulent kinetic energy k and its dissipation rate ε. And an improved mixed turbulence model containing the turbulent kinetic energy k, dissipation rate ε and the mean field velocity strain rate is proposed. Then the model is used to simulate the turbulent flow over backward-facing step, and the calculated distributions of longitudinal velocity, turbulent kinetic energy k and dissipation rate ε are compared with the experimental data and the similar simulations of the standard k-ε model. The analysis shows that the longitudinal velocity distribution calculated by the improved mixed turbulence model is closer to the experimental values than that of the k-ε model. In addition, the improved mixed turbulence model cannot only calculate the distribution of turbulent kinetic energy k near the wall under the condition of large Reynolds number better, but also successfully simulate the additional separation vortex downstream the main separation zone, which is an important progress. What's more, the improved model can simulate the distribution of kinetic energy dissipation rate ε well. Therefore, the improved mixed turbulence model has higher accuracy for simulation of such turbulent flow. This will be of great significance to the study of complex turbulence characteristics and computation of turbulent flow with vortex separation in the future.

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